

Circles: Angles, Arcs, Chords and Tangents

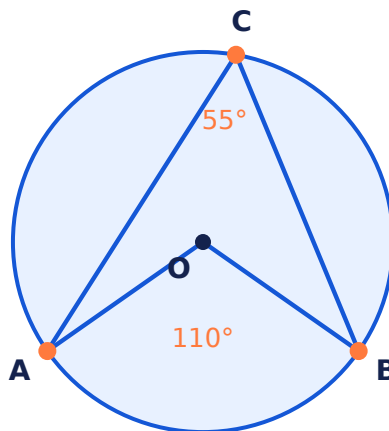
Grade 10 – Geometry math · Lesson · Standards: G-C.A.1, G-C.A.2

Quick answer: Angles in a circle are tied to arcs: a central angle equals its intercepted arc, while an inscribed angle is half of it. An angle inscribed in a semicircle measures 90° , opposite angles of an inscribed quadrilateral add to 180° , and a tangent is perpendicular to the radius.

Standards: G-C.A.1, G-C.A.2 · **Level:** Grade 10 Geometry

What is the difference between a central angle and an inscribed angle?

A central angle has its vertex at the center of the circle. An inscribed angle has its vertex on the circle itself. Both can open onto the same arc, and then a simple pattern appears.



Arc AB = 110° , central angle = 110° , inscribed angle = 55° .

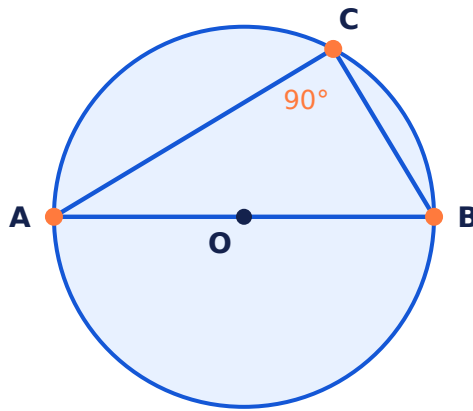
Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc. A central angle is equal to its arc.

$$m\angle ACB = \frac{1}{2} m(\text{arc } AB) = \frac{1}{2} m\angle AOB$$

What special cases are worth remembering?

- Inscribed angles that intercept the same arc are congruent.
- An angle inscribed in a semicircle is a right angle, because its arc is 180° .
- Opposite angles of a quadrilateral inscribed in a circle are supplementary.

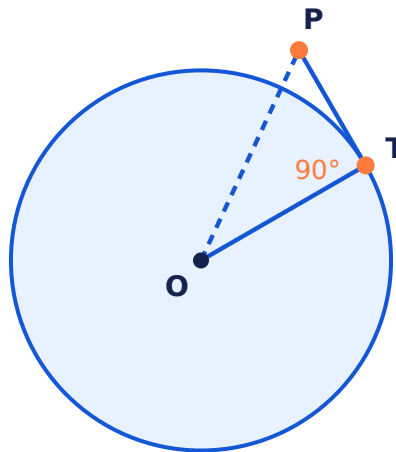


Diameter AB: the angle at C is always 90° .

How do tangents and chords behave?

Tangent and chord facts

- A tangent is perpendicular to the radius drawn to the point of tangency.
- Two tangent segments from the same outside point are equal in length.
- A radius perpendicular to a chord bisects the chord, and congruent chords in one circle cut off congruent arcs.



Right angle at T: $OP^2 = OT^2 + TP^2$.

Method for angle problems

1. Mark every central, inscribed or tangent angle and the arc it opens onto.
2. Translate each angle to its arc: double an inscribed angle, keep a central angle.
3. Use 360° around the circle, 180° in a triangle or a straight line, and 90° at a tangent.
4. Convert back to the angle you need and check it against the figure.

Example

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Arc AB is 88° and arc BC is 126° . Find the inscribed angle at a point D on the major arc that opens onto arc AC through B .

The arc from A through B to C measures $88^\circ + 126^\circ = 214^\circ$. The inscribed angle is half of it: **107°** .

Common mistake

Using the whole arc as the inscribed angle. An inscribed angle is half of the arc it intercepts.

Mathbot's tip When a diameter shows up, look for a hidden 90° angle at the circle. When a tangent shows up, draw the radius to the point of tangency.

Key points to remember

- Central angle = intercepted arc; inscribed angle = $\frac{1}{2}$ intercepted arc.
- Inscribed angles on the same arc are congruent.
- Opposite angles of an inscribed quadrilateral are supplementary.
- A tangent line is perpendicular to the radius at the point of tangency.

Circles: Angles, Arcs, Chords and Tangents: frequently asked questions

Why is an inscribed angle half of the central angle on the same arc?

Draw a radius to the vertex to make isosceles triangles. The exterior angle of each triangle equals the sum of two equal base angles, so the central angle is twice the inscribed angle.

Why are all circles similar?

A translation moves one center onto the other, and a dilation scaled by the ratio of the radii then stretches one circle exactly onto the other. A similarity transformation exists for any two circles.

What is an intercepted arc?

It is the part of the circle that lies in the interior of an angle, between the points where the sides of the angle meet the circle.

How do I find the angle between two tangents?

Subtract the central angle from 180° , or take half the difference of the major and minor arcs. The two radii and two tangents form a quadrilateral with two right angles.

Why are opposite angles of an inscribed quadrilateral supplementary?

Each of the two opposite angles intercepts one of two arcs that together make the full 360° circle.

Half of 360° is 180° .