

Circles: Angles, Arcs, Chords and Tangents

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-C.A.1, G-C.A.2

Each solution names the circle theorem before using it. Check that the arcs of a full circle add to 360° and that opposite angles of an inscribed quadrilateral add to 180° before accepting an answer.

Standards: G-C.A.1, G-C.A.2 · **Level:** Grade 10 Geometry

Problem 1 Central Angle and Arc

A central angle has the same measure as its intercepted arc.

$$m\angle AOB = 118^\circ$$

Problem 2 Inscribed Angle from an Arc

The inscribed angle is half of its arc: $\frac{1}{2} \cdot 134^\circ = 67^\circ$.

$$m\angle ACB = 67^\circ$$

Problem 3 Arc from an Inscribed Angle

The arc is twice the inscribed angle: $2 \cdot 38^\circ = 76^\circ$.

$$76^\circ$$

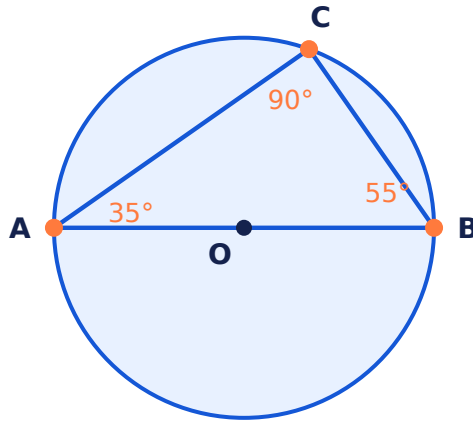
Problem 4 Why Circles Are Similar

(a) Translate circle A so that its center lands on the center of B, then dilate about that center with scale factor $1\frac{5}{6} = \frac{5}{2}$.

(b) Circumferences scale by the factor $\frac{5}{2}$ ($30\pi \div 12\pi$). Areas scale by the square: $2\frac{5}{4}$ ($225\pi \div 36\pi$).

(a) translation, then dilation with factor 5/2 (b) 5 : 2 and 25 : 4

Problem 5 Angle in a Semicircle



Solved angles.

An angle inscribed in a semicircle is a right angle, so $m\angle C = 90^\circ$.

$$m\angle B = 180^\circ - 90^\circ - 35^\circ = 55^\circ.$$

$$\angle C = 90^\circ, \angle B = 55^\circ$$

Problem 6 Tangent Length

$OT \perp PT$, so triangle OTP is a right triangle with hypotenuse OP .

$$OP = \sqrt{(8^2 + 15^2)} = \sqrt{289} = 17.$$

$$\mathbf{OP = 17 \text{ cm}}$$

Problem 7 Angle Between Two Tangents

Quadrilateral $OAPB$ has right angles at A and B , so $\angle P + \angle AOB = 180^\circ$.

$$m\angle P = 180^\circ - 118^\circ = 62^\circ.$$

$$\mathbf{m\angle P = 62^\circ}$$

Problem 8 Inscribed Quadrilateral Angles

Opposite angles are supplementary. $\angle C = 180^\circ - 95^\circ = 85^\circ$. $\angle D = 180^\circ - 72^\circ = 108^\circ$.

Check: $95 + 72 + 85 + 108 = 360$.

$$\mathbf{\angle C = 85^\circ, \angle D = 108^\circ}$$

Problem 9 Solving for x with an Arc

Inscribed angle = $\frac{1}{2}$ arc: $4x + 3 = \frac{1}{2}(12x - 18) = 6x - 9$.

$12 = 2x$, so $x = 6$. The angle is $4(6) + 3 = 27^\circ$ and the arc is 54° .

$x = 6$; angle = 27°

Problem 10 Angles on the Same Arc

Inscribed angles on the same arc are equal: $5x - 12 = 3x + 14$, so $2x = 26$ and $x = 13$.

The angle is $5(13) - 12 = 53^\circ$.

$x = 13$; angle = 53°

Problem 11 Congruent Chords

Congruent chords cut off congruent arcs: $7x + 9 = 9x - 17$, so $26 = 2x$ and $x = 13$.

Arc $AB = 7(13) + 9 = 100^\circ$. Check: arc $CD = 9(13) - 17 = 100^\circ$.

$x = 13$; each arc is 100°

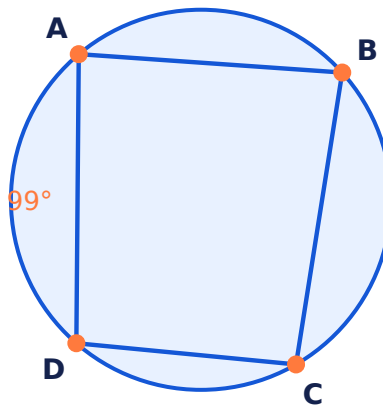
Problem 12 Distance from the Center to a Chord

The perpendicular from the center bisects the chord, so a right triangle has legs 7 and d and hypotenuse 11.

$$d = \sqrt{(11^2 - 7^2)} = \sqrt{72} = 6\sqrt{2} \approx 8.49.$$

$6\sqrt{2} \approx 8.5$ in

Problem 13 Four Arcs Around a Circle



$\text{Arc } DA = 99^\circ.$

$\text{Arc } DA = 360^\circ - (88^\circ + 102^\circ + 71^\circ) = 99^\circ.$

$\angle ABC$ intercepts arc $ADC = 71^\circ + 99^\circ = 170^\circ$, so $\angle ABC = 85^\circ$.

$\angle ADC$ intercepts arc $ABC = 88^\circ + 102^\circ = 190^\circ$, so $\angle ADC = 95^\circ$.

$85^\circ + 95^\circ = 180^\circ$, as expected for opposite angles of an inscribed quadrilateral.

$DA = 99^\circ$, $\angle ABC = 85^\circ$, $\angle ADC = 95^\circ$, sum 180°

Problem 14 Tangents with a Major Arc

Minor arc $= 360^\circ - 238^\circ = 122^\circ$.

$m\angle P = \frac{1}{2}(238^\circ - 122^\circ) = \frac{1}{2}(116^\circ) = 58^\circ$. Check: $180^\circ - 122^\circ = 58^\circ$.

Minor arc $= 122^\circ$, $\angle P = 58^\circ$

Problem 15 Guard at the Pond

The tangent segment is a leg of a right triangle with hypotenuse 22 and other leg 9: $\sqrt{(22^2 - 9^2)} = \sqrt{403} \approx 20.07$.

Half the angle at P satisfies $\sin(\frac{1}{2}\angle P) = \frac{9}{22}$, so $\frac{1}{2}\angle P \approx 24.15^\circ$ and $\angle P \approx 48.3^\circ$.

Tangent ≈ 20.1 ft; angle $\approx 48.3^\circ$

Problem 16 Triangle in a Window

(a) AB is a diameter, so the inscribed angle at C intercepts a semicircle of 180° and measures half: 90° .

$$(b) BC = \sqrt{(29^2 - 20^2)} = \sqrt{441} = 21.$$

$$(c) \tan A = \frac{21}{20}, \text{ so } \angle A = 46.4^\circ \approx 46.4^\circ.$$

(b) BC = 21 in (c) $\angle A \approx 46.4^\circ$

Problem 17 Lights on a Circular Track

Opposite angles are supplementary. $(6x + 4) + (4x + 16) = 180$ gives $10x = 160$, so $x = 16$. $\angle A = 100^\circ$, $\angle C = 80^\circ$.

$$(7y - 5) + (3y + 25) = 180 \text{ gives } 10y = 160, \text{ so } y = 16. \angle B = 107^\circ, \angle D = 73^\circ.$$

$$\text{Check: } 100 + 80 + 107 + 73 = 360.$$

A = 100°, B = 107°, C = 80°, D = 73°

Problem 18 Roundabout Tangent Roads

The line from P to the center bisects the 54° angle, giving 27° , and the radius to the tangent point is perpendicular to the road.

$$OP = \frac{60}{(\sin 27^\circ)} \approx 132.16 \text{ ft. Road length} = \frac{60}{(\tan 27^\circ)} \approx 117.76 \text{ ft.}$$

OP $\approx$ 132.2 ft; each road $\approx$ 117.8 ft

Problem 19 Clock Face Triangle

Adjacent hour marks are 30° apart. Arc 12 to 4 = 120° , arc 4 to 9 = 150° , arc 9 to 12 = 90° .

The angle at 12 intercepts the arc from 4 to 9: $\frac{1}{2} \cdot 150^\circ = 75^\circ$. The angle at 4 intercepts the arc from 9 to 12: $\frac{1}{2} \cdot 90^\circ = 45^\circ$. The angle at 9 intercepts the arc from 12 to 4: $\frac{1}{2} \cdot 120^\circ = 60^\circ$.

$$75 + 45 + 60 = 180.$$

75° at 12, 45° at 4, 60° at 9

Problem 20 Triangle from Three Arcs

$\angle C$ intercepts arc AB : 50° . $\angle A$ intercepts arc BC : 75° . $\angle B$ intercepts arc CA : 55° . Sum: 180° .

The longest side is opposite the largest angle, $\angle A$, so it is BC .

A = 75°, B = 55°, C = 50°; longest side BC