

Proving Triangles Congruent: SSS to HL

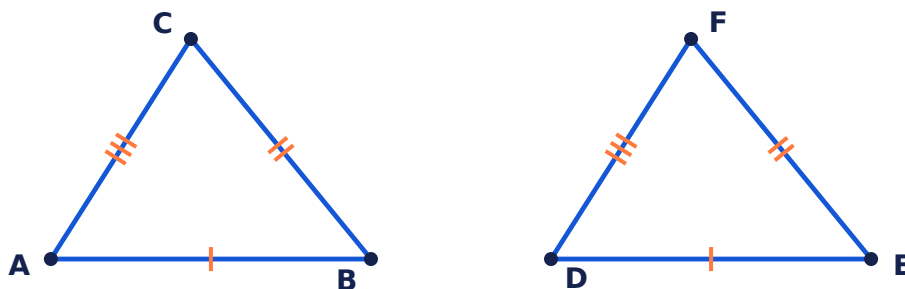
Grade 10 – Geometry math · Lesson · Standards: G-CO.B.6, G-CO.B.7, G-CO.B.8, G-SRT.B.5

Quick answer: Two triangles are congruent when rigid motions map one onto the other, so all corresponding sides and angles match. Triangles can be proved congruent with SSS, SAS, ASA, AAS or, for right triangles, HL. SSA and AAA do not work. After proving congruence, CPCTC says every remaining pair of corresponding parts is congruent.

Standards: G-CO.B.6, G-CO.B.7, G-CO.B.8, G-SRT.B.5 · **Level:** Grade 10 Geometry

What does congruent mean for triangles?

Two triangles are **congruent** when a sequence of rigid motions (translations, reflections and rotations) carries one exactly onto the other. Then all six **corresponding parts**, three sides and three angles, match. The order of letters in a congruence statement shows the matching: $\triangle ABC \cong \triangle DEF$ means $\angle A \cong \angle D$, $AB \cong DE$, and so on.

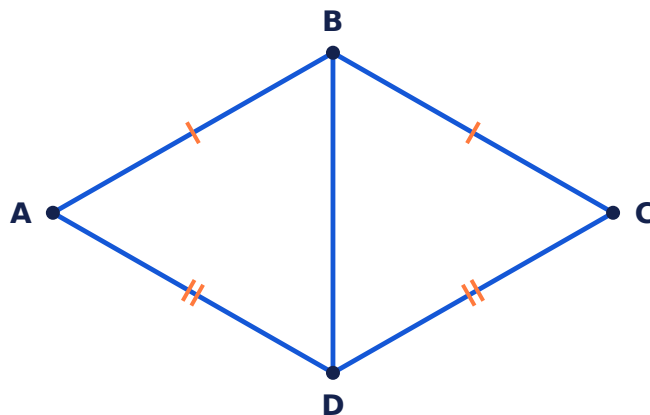


Three pairs of congruent sides prove $\triangle ABC \cong \triangle DEF$ by SSS.

Which shortcuts prove that triangles are congruent?

You do not need all six parts. Three well-chosen parts are enough.

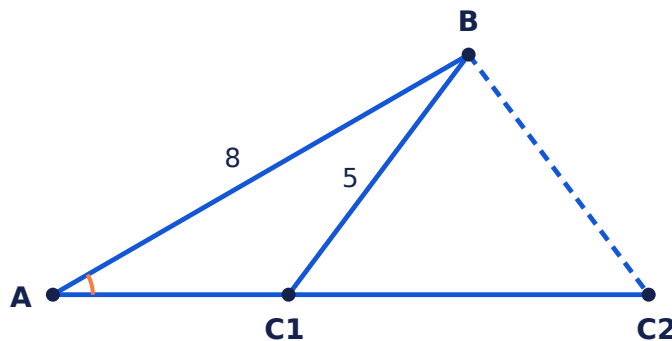
Criterion	Parts that match	Note
SSS	Three sides	No angles needed
SAS	Two sides and the angle between them	Angle must be included
ASA	Two angles and the side between them	Side must be included
AAS	Two angles and a side not between them	Follows from ASA
HL	Hypotenuse and one leg of right triangles	Right triangles only



$\triangle ABD$ and $\triangle CBD$ share side BD , so SSS applies.

Why does SSA fail?

Two sides and an angle that is *not* between them can build two different triangles. With $\angle A = 30^\circ$, $AB = 8$ and $BC = 5$, side BC can swing to two spots on the ray from A , so AC may be about 3.9 or about 9.9. AAA also fails: it fixes shape but not size.



Same angle, same two sides, two different triangles: SSA does not prove congruence.

How do you write a congruence proof?

Method

1. Mark the Given facts on the figure, including shared sides and vertical angles.
2. Pick a criterion: SSS, SAS, ASA, AAS or HL.
3. List three congruent pairs, each with a reason.
4. State that the triangles are congruent by that criterion.
5. Use **CPCTC** (corresponding parts of congruent triangles are congruent) to reach the Prove statement.

Worked example

Given: $AB \cong CB$ and $AD \cong CD$ in the kite above. Prove: $\angle A \cong \angle C$. Since $BD \cong BD$ (reflexive), $\triangle ABD \cong \triangle CBD$ by SSS. By CPCTC, $\angle A \cong \angle C$.

Common mistake

Do not call a pair of triangles congruent by SAS when the angle sits outside the two marked sides. The angle must be between them.

Mathbot's tip

Name the triangles with vertices in matching order. A correct statement like $\triangle ABC \cong \triangle DEF$ tells you every corresponding pair at once.

Key points to remember

- Congruent triangles have three pairs of congruent sides and three pairs of congruent angles.
- Valid criteria: SSS, SAS, ASA, AAS and HL for right triangles.
- SSA can describe two different triangles, so it does not prove congruence.
- CPCTC lets you conclude that remaining corresponding parts are congruent.
- A shared side is congruent to itself by the Reflexive Property.

Proving Triangles Congruent: SSS to HL: frequently asked questions

What does CPCTC stand for?

CPCTC means "corresponding parts of congruent triangles are congruent." You use it only after you have proved the triangles congruent.

How do you choose between ASA and AAS?

Look at where the marked side sits. If it is between the two marked angles, use ASA. If it is next to only one of them, use AAS.

Why is HL allowed when SSA is not?

In a right triangle the angle opposite the hypotenuse is 90° , which removes the ambiguity. The remaining leg is forced by the Pythagorean Theorem.

Does AAA prove triangles congruent?

No. Triangles with equal angles have the same shape but can be different sizes, so they are similar and not necessarily congruent.

How do rigid motions define congruence?

If a translation, reflection, rotation or a sequence of them moves one triangle exactly onto another, the triangles are congruent. Rigid motions preserve every length and angle.