

Proving Triangles Congruent: SSS to HL

Grade 10 – Geometry math · Test Answer Key · Standards: G-CO.B.6, G-CO.B.7, G-CO.B.8, G-SRT.B.5

Each solution gives the reasons and the points awarded. A proof earns partial credit for correct congruent pairs even if the final criterion is missing.

Standards: G-CO.B.6, G-CO.B.7, G-CO.B.8, G-SRT.B.5 · **Level:** Grade 10 Geometry

Question 1 Corresponding parts with algebra (2 points)

- a. $4x - 3 = 2x + 9$ gives $x = 6$, so $DE = 2(6) + 9 = 21$.
- b. $7y - 6 = 4y + 18$ gives $3y = 24$ and $y = 8$, so $m\angle F = 4(8) + 18 = 50^\circ$.

(a) DE = 21, (b) 50°

Scoring: 1 point per part.

Question 2 Choose the criterion (3 points)

- a. $\angle B$ lies between AB and BC : SAS.
- b. AC lies between $\angle A$ and $\angle C$: ASA.
- c. The angle is not between the two sides: SSA, so none.

(a) SAS, (b) ASA, (c) none

Scoring: 1 point per part.

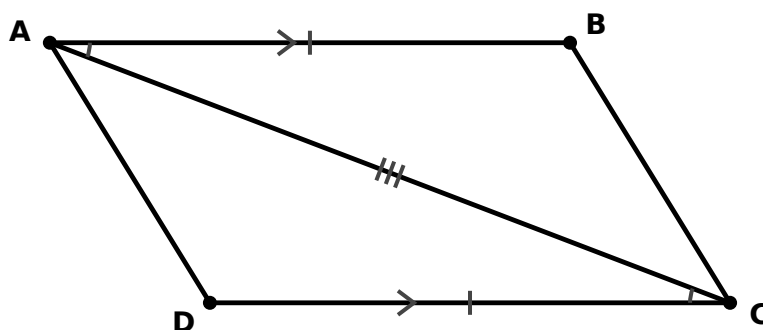
Question 3 Right triangles (3 points)

- a. Hypotenuse and a leg of right triangles: HL.
- b. $BC = \sqrt{(15^2 - 9^2)} = \sqrt{144} = 12$.
- c. Area = $\frac{1}{2} \cdot 9 \cdot 12 = 54$, using the congruent leg $EF = 12$.

(a) HL, (b) BC = 12, (c) 54 square units

Scoring: 1 point per part.

Question 4 Prove with SAS (3 points)



Solved figure with the alternate interior angles and the shared diagonal marked.

Statement	Reason
1. $AB \parallel DC$ and $AB \cong DC$	Given
2. $\angle BAC \cong \angle DCA$	Alternate interior angles (parallel lines)
3. $AC \cong CA$	Reflexive Property
4. $\triangle ABC \cong \triangle CDA$	SAS

Scoring: 1 point for step 2 with reason, 1 point for step 3, 1 point for the SAS conclusion.

Question 5 Congruence on the plane (3 points)

$\triangle RST$: $RS = 6$, $ST = 8$, $RT = \sqrt{6^2 + 8^2} = 10$.

$\triangle UVW$: $UV = 7 - 1 = 6$, $VW = 9 - 1 = 8$, $UW = \sqrt{8^2 + 6^2} = 10$.

The side lengths 6, 8, 10 match, so $\triangle RST \cong \triangle UVW$ by SSS.

Congruent by SSS

Scoring: 1 point for each triangle's side lengths, 1 point for the SSS conclusion.

Question 6 Why SSA fails (2 points)

No. The given parts form an SSA pattern. The side of length 7 can meet the base ray in two different places, producing two different triangles with the same three measurements.

No, SSA does not prove congruence

Scoring: 1 point for "no," 1 point for a valid explanation.

Question 7 Truss beams (2 points)

$DF \cong CA$, so $DF = 5.6$ m. One triangle uses $3.5 + 4.2 + 5.6 = 13.3$ m of beam; two use 26.6 m.
Cost = $26.6 \times 12.50 = \$332.50$.

DF = 5.6 m; cost \$332.50

Scoring: 1 point for DF, 1 point for the cost.

Question 8 Paragraph proof with ASA (2 points)

Because BD bisects $\angle ABC$, $\angle ABD \cong \angle CBD$. Because BD bisects $\angle ADC$, $\angle ADB \cong \angle CDB$. The side BD is shared, so $BD \cong BD$ by the Reflexive Property. The shared side lies between the congruent angles, so $\triangle ABD \cong \triangle CBD$ by ASA. Then $AB \cong CB$ by CPCTC.

$AB \cong CB$

Scoring: 1 point for the ASA argument with all three pairs, 1 point for the CPCTC conclusion.