

Proving Triangles Congruent: SSS to HL

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-CO.B.6, G-CO.B.7, G-CO.B.8, G-SRT.B.5

Each answer matches corresponding parts, names the criterion and finishes with CPCTC when a part of a triangle must be proved. Check that every congruence statement lists vertices in matching order.

Standards: G-CO.B.6, G-CO.B.7, G-CO.B.8, G-SRT.B.5 · **Level:** Grade 10 Geometry

Problem 1 Corresponding parts

The letters correspond in order: $P \leftrightarrow X$, $Q \leftrightarrow Y$, $R \leftrightarrow Z$. So $XY = PQ = 9.5$, $YZ = QR = 12$, $m\angle X = 51^\circ$ and $m\angle Y = 77^\circ$.

$m\angle R = 180^\circ - 51^\circ - 77^\circ = 52^\circ$, so $m\angle Z = 52^\circ$.

$XY = 9.5$, $YZ = 12$, $\angle X = 51^\circ$, $\angle Y = 77^\circ$, $\angle Z = 52^\circ$

Problem 2 Name the criterion

- a. SSS.
- b. SAS: the angle is included.
- c. ASA: the side is included.
- d. AAS: the side is not between the angles.
- e. HL.

(a) SSS, (b) SAS, (c) ASA, (d) AAS, (e) HL

Problem 3 Why SSA is not enough

No. The angle is not included, so this is SSA. The side of length 6 can reach the base ray in two places, so two different triangles can have these measurements. SSA is not a congruence criterion.

No: SSA does not guarantee congruence

Problem 4 Solve for x

Corresponding sides are equal: $4x + 1 = 7x - 11$, so $12 = 3x$ and $x = 4$.

$AB = 4(4) + 1 = 17$, and $DE = 7(4) - 11 = 17$.

$x = 4$; $AB = 17$

Problem 5 Shared side

Both triangles contain side BD , and $BD \cong BD$ by the Reflexive Property. Three pairs of sides match.

$BD \cong BD$; $\triangle ABD \cong \triangle CBD$ by SSS

Problem 6 Read the marks

The marked parts are $AB \cong DE$, $\angle A \cong \angle D$ and $AC \cong DF$. The angle at A lies between sides AB and AC , so it is included.

SAS

Problem 7 Angle sum in congruent triangles

$\angle A \cong \angle D$: $4x + 3 = 6x - 17$, so $20 = 2x$ and $x = 10$. Then $m\angle A = 43^\circ$.

$\angle C = 180^\circ - 43^\circ - 62^\circ = 75^\circ$, and $\angle F \cong \angle C$.

$x = 10$; $m\angle F = 75^\circ$

Problem 8 Perimeter of congruent triangles

$JK \cong MN$: $2x + 1 = 13$, so $x = 6$. Check with $KL \cong NP$: $3(6) - 2 = 16 \checkmark$.

$JL = 4(6) - 5 = 19$. Perimeter = $13 + 16 + 19 = 48$.

$x = 6$; $JL = 19$; perimeter 48

Problem 9 ASA or AAS

- AB lies between $\angle A$ and $\angle B$: ASA.
- BC is not between $\angle A$ and $\angle B$: AAS.
- Three pairs of angles (AAA) fix only the shape, not the size: no criterion applies.

(a) ASA, (b) AAS, (c) not enough

Problem 10 Hypotenuse and leg

The hypotenuses are congruent ($AC = DF = 13$) and one pair of legs is congruent ($AB = DE = 5$), so $\triangle ABC \cong \triangle DEF$ by HL.

By the Pythagorean Theorem, $BC = \sqrt{(13^2 - 5^2)} = \sqrt{144} = 12$, and $EF = 12$ as a corresponding side.

HL; $BC = EF = 12$

Problem 11 Congruence from coordinates

Find the side lengths. $\triangle ABC$: $AB = 4$ (horizontal), $BC = 3$ (vertical), $AC = \sqrt{(4^2 + 3^2)} = 5$.

$\triangle DEF$: $DE = |-2 - (-6)| = 4$, $EF = |-3 - (-6)| = 3$, $DF = \sqrt{(3^2 + 4^2)} = 5$.

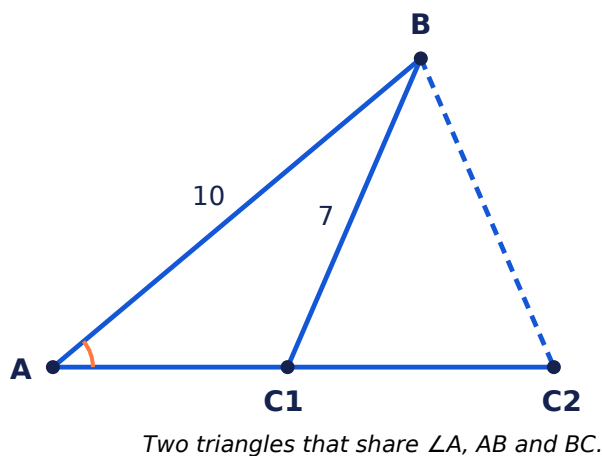
Matching sides 4, 3, 5 gives $\triangle ABC \cong \triangle DEF$ by SSS.

Congruent by SSS (sides 4, 3, 5)

Problem 12 The ambiguous SSA case

Since $6.4 < 7 < 10$, the circle of radius 7 about B meets the ray from A in two points. The foot of the height is $10 \cos 40^\circ \approx 7.66$ from A , and the offset along the ray is $\sqrt{(7^2 - 6.43^2)} \approx 2.77$.

So $AC \approx 7.66 - 2.77 \approx 4.89$ or $7.66 + 2.77 \approx 10.43$.



Two triangles: $AC \approx 4.9$ or $AC \approx 10.4$

Problem 13 Prove angles congruent with SSS

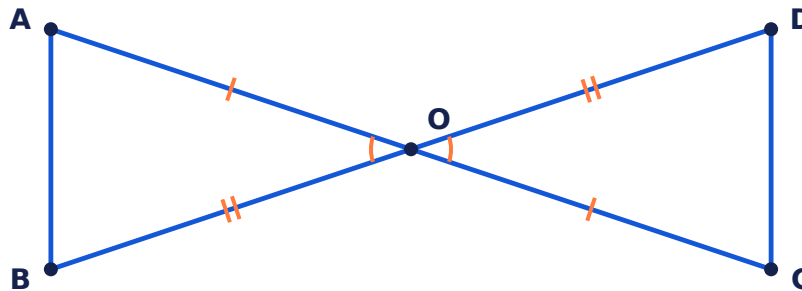
Statement	Reason
1. $AB \cong CB$ and $AD \cong CD$	Given

Statement	Reason
2. $BD \cong BD$	Reflexive Property
3. $\triangle ABD \cong \triangle CBD$	SSS
4. $\angle BAD \cong \angle BCD$	CPCTC

$$\angle BAD \cong \angle BCD$$

Problem 14 Crossing midpoints

Statement	Reason
1. O is the midpoint of AC and of BD	Given
2. $AO \cong CO$ and $BO \cong DO$	Definition of midpoint
3. $\angle AOB \cong \angle COD$	Vertical angles are congruent
4. $\triangle AOB \cong \triangle COD$	SAS



Solved figure: the vertical angles at O are marked as congruent.

$$\triangle AOB \cong \triangle COD \text{ by SAS}$$

Problem 15 Roof truss braces

$PQ = PS = 18$, $QR = SR = 14.5$, and PR is shared. Three pairs of sides match, so $\triangle PQR \cong \triangle PSR$ by SSS.

$\angle PQR$ corresponds to $\angle PSR$, so by CPCTC $m\angle PQR = 112^\circ$.

Congruent by SSS; $m\angle PQR = 112^\circ$

Problem 16 Matching ramps

Both are right triangles with equal hypotenuses (10 ft) and a pair of equal legs (6 ft), so they are congruent by HL.

Height = $\sqrt{(10^2 - 6^2)} = \sqrt{64} = 8$ feet. Triangular side area = $\frac{1}{2} \cdot 6 \cdot 8 = 24$ square feet.

Congruent by HL; height 8 ft; area 24 ft²

Problem 17 Bisector and perpendicular

Statement	Reason
1. BD bisects $\angle ABC$	Given
2. $\angle ABD \cong \angle CBD$	Definition of angle bisector
3. $BD \cong BD$	Reflexive Property
4. $BD \perp AC$	Given
5. $\angle BDA \cong \angle BDC$	Perpendicular lines form congruent right angles
6. $\triangle ABD \cong \triangle CBD$	ASA (side BD is between the marked angles)
7. $AD \cong CD$	CPCTC

$AD \cong CD$

Problem 18 Prove sides congruent with AAS

Statement	Reason
1. $\angle B \cong \angle E$, $\angle C \cong \angle F$, $AB \cong DE$	Given
2. $\triangle ABC \cong \triangle DEF$	AAS (side AB is not between $\angle B$ and $\angle C$)
3. $AC \cong DF$	CPCTC

$AC \cong DF$

Problem 19 Congruent triangles with algebra

Match corresponding sides: $AB = XY$ gives $3x - 1 = 17$, so $x = 6$. Check: $BC = 2(6) + 5 = 17$ ✓ and $CA = 6 + 12 = 18$ ✓.

Perimeter = $17 + 17 + 18 = 52$. Two sides are equal, so the triangle is isosceles.

$x = 6$; perimeter 52; isosceles

Problem 20 Overlapping triangles

Statement	Reason
1. $AB \cong AC$ and $AD \cong AE$	Given
2. $\angle BAE \cong \angle CAD$	Both are the same angle $\angle A$ (Reflexive Property)
3. $\triangle ABE \cong \triangle ACD$	SAS ($\angle A$ is between the marked sides)
4. $BE \cong CD$	CPCTC

$BE \cong CD$