

Proving Triangles Congruent: SSS to HL

Grade 10 – Geometry math · Worksheet · Standards: G-CO.B.6, G-CO.B.7, G-CO.B.8, G-SRT.B.5

Name: _____ Date: _____ Class: _____

These problems cover G-CO.B.6, G-CO.B.7, G-CO.B.8 and G-SRT.B.5: congruence through rigid motions, corresponding parts, the SSS, SAS, ASA, AAS and HL criteria, the failure of SSA, and proofs that finish with CPCTC.

Early problems match parts and name criteria. Later problems ask for two-column proofs with shared sides and overlapping triangles.

Standards: G-CO.B.6, G-CO.B.7, G-CO.B.8, G-SRT.B.5 · **Level:** Grade 10 Geometry

Problem 1 Corresponding parts (easy)

$\triangle PQR \cong \triangle XYZ$, with $PQ = 9.5$, $QR = 12$, $m\angle P = 51^\circ$ and $m\angle Q = 77^\circ$. Find XY , YZ , $m\angle X$, $m\angle Y$ and $m\angle Z$.

Problem 2 Name the criterion (easy)

Name the congruence criterion (SSS, SAS, ASA, AAS, HL or none) that each description supports.

- Three pairs of sides are congruent.
- Two pairs of sides and the angles between them are congruent.
- Two pairs of angles and the sides between them are congruent.
- Two pairs of angles and a pair of sides that is not between them are congruent.
- Two right triangles have congruent hypotenuses and one pair of congruent legs.

Problem 3 Why SSA is not enough (easy)

Two triangles each have a side of 6, a side of 9 and a 40° angle that is not between those sides. A student says they must be congruent. Is the student right?

Problem 4 Solve for x (easy)

$\triangle ABC \cong \triangle DEF$. $AB = 4x + 1$ and $DE = 7x - 11$. Find x and AB .

Problem 5 Shared side (easy)

In the figure, $AB \cong CB$ and $AD \cong CD$. Name a third pair of congruent parts and the criterion that proves $\triangle ABD \cong \triangle CBD$.

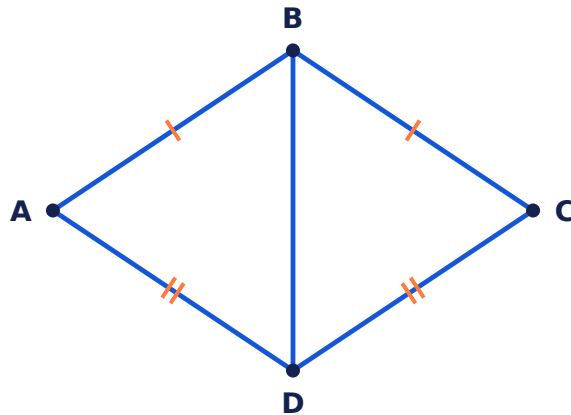


Figure for Problem 5.

Problem 6 Read the marks (medium)

Use the marks to decide which criterion proves the triangles congruent.

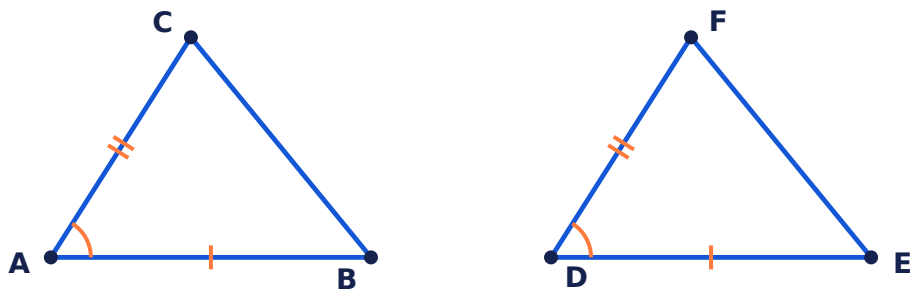


Figure for Problem 6.

Problem 7 Angle sum in congruent triangles (medium)

$\triangle ABC \cong \triangle DEF$, with $m\angle A = (4x + 3)^\circ$, $m\angle D = (6x - 17)^\circ$ and $m\angle B = 62^\circ$. Find x and $m\angle F$.

Problem 8 Perimeter of congruent triangles (medium)

$\triangle JKL \cong \triangle MNP$. $JK = 2x + 1$, $KL = 3x - 2$, $JL = 4x - 5$, $MN = 13$ and $NP = 16$. Find x , JL and the perimeter of $\triangle JKL$.

Problem 9 ASA or AAS (medium)

In $\triangle ABC$ and $\triangle DEF$, decide whether each set of givens proves congruence, and name the criterion.

- a. $\angle A \cong \angle D, AB \cong DE, \angle B \cong \angle E$
- b. $\angle A \cong \angle D, \angle B \cong \angle E, BC \cong EF$
- c. $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$

Problem 10 Hypotenuse and leg (medium)

$\triangle ABC$ and $\triangle DEF$ are right triangles with right angles at B and E . $AC = DF = 13$ and $AB = DE = 5$. Prove the triangles are congruent, then find BC and EF .

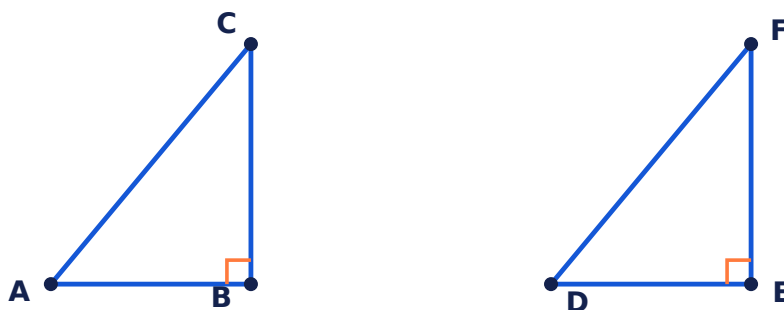


Figure for Problem 10.

Problem 11 Congruence from coordinates (medium)

$\triangle ABC$ has $A(1, 1)$, $B(5, 1)$ and $C(5, 4)$. $\triangle DEF$ has $D(-3, -2)$, $E(-3, -6)$ and $F(-6, -6)$. Show that the triangles are congruent.

Problem 12 The ambiguous SSA case (medium)

In a triangle, $\angle A = 40^\circ$, $AB = 10$ and $BC = 7$. The height from B to the line through A and C is $10 \sin 40^\circ \approx 6.4$. Explain why two non-congruent triangles fit, and estimate both lengths of AC .

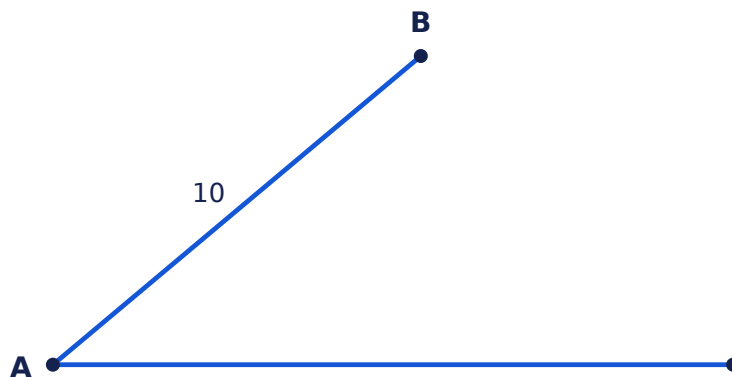


Figure for Problem 12 (point C not yet placed).

Problem 13 Prove angles congruent with SSS (medium)

Given: $AB \cong CB$ and $AD \cong CD$. Prove: $\angle BAD \cong \angle BCD$. Write a two-column proof.

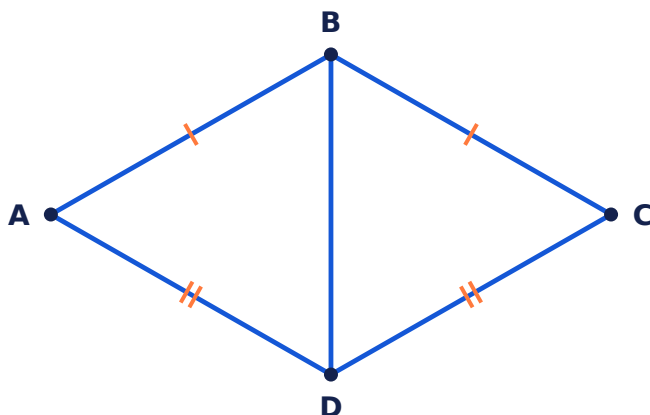


Figure for Problem 13.

Problem 14 Crossing midpoints (medium)

Segments AC and BD intersect at O , and O is the midpoint of both. Prove: $\triangle AOB \cong \triangle COD$.

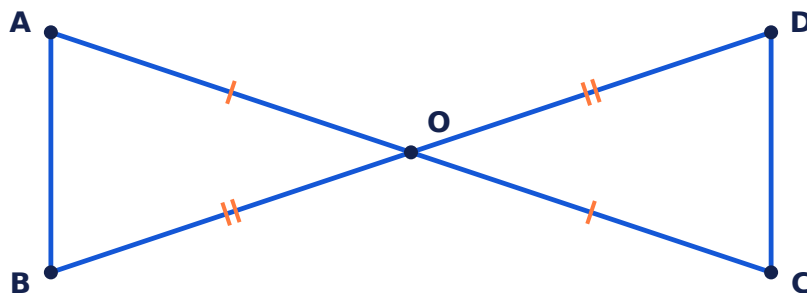


Figure for Problem 14.

Problem 15 Roof truss braces (hard)

A roof truss has two triangular braces, $\triangle PQR$ and $\triangle PSR$, sharing the beam PR . $PQ = PS = 18$ feet and $QR = SR = 14.5$ feet. Show that the braces are congruent, and find $m\angle PQR$ when $m\angle PSR = 112^\circ$.

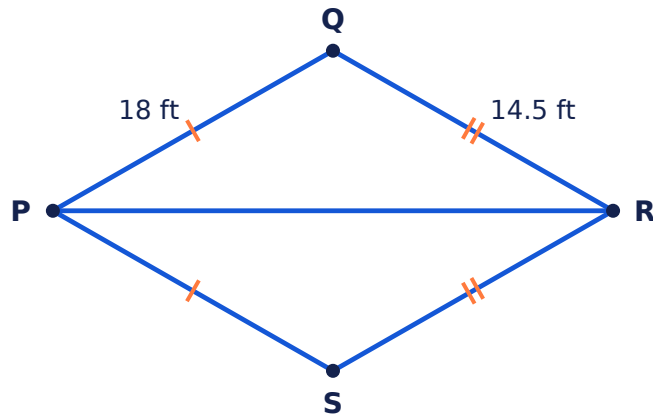


Figure for Problem 15.

Problem 16 Matching ramps (hard)

A skate park builds two right-triangle ramps. Each ramp has a hypotenuse of 10 feet and a base leg of 6 feet. Show the ramps are congruent, find the height of each, and find the area of one triangular side.

Problem 17 Bisector and perpendicular (hard)

Given: BD bisects $\angle ABC$ and $BD \perp AC$ at D . Prove: $AD \cong CD$. Write a two-column proof.

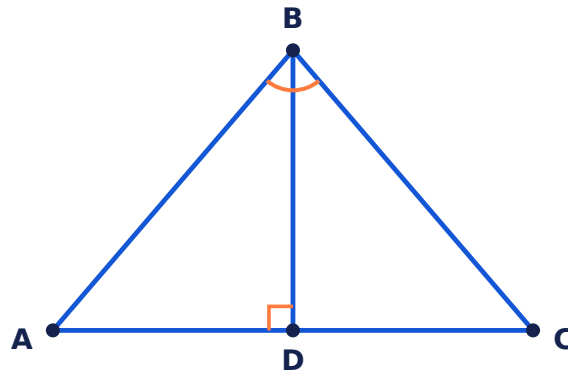


Figure for Problem 17.

Problem 18 Prove sides congruent with AAS (hard)

Given: $\angle B \cong \angle E$, $\angle C \cong \angle F$ and $AB \cong DE$. Prove: $AC \cong DF$. Write a two-column proof.

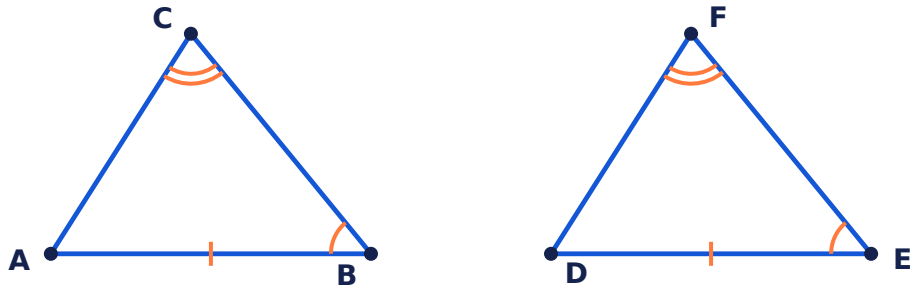


Figure for Problem 18.

Problem 19 Congruent triangles with algebra (hard)

$\triangle ABC \cong \triangle XYZ$. The sides of $\triangle ABC$ are $AB = 3x - 1$, $BC = 2x + 5$ and $CA = x + 12$. In $\triangle XYZ$, $XY = 17$, $YZ = 17$ and $XZ = 18$. Find x , the perimeter, and say what type of triangle this is by its sides.

Problem 20 Overlapping triangles (hard)

Points D and E lie on AB and AC . Given: $AB \cong AC$ and $AD \cong AE$. Prove: $BE \cong CD$. Write a two-column proof.

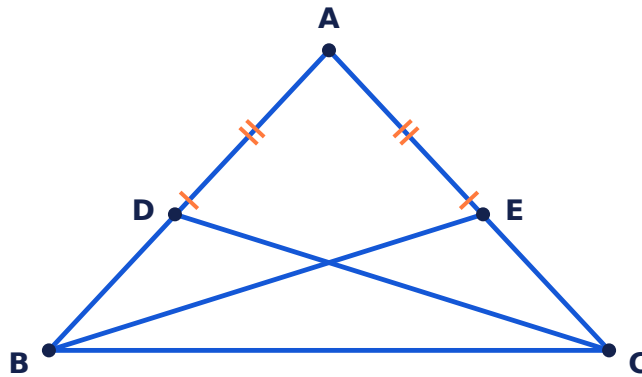


Figure for Problem 20.