

Proving Shapes with Coordinates and Slopes

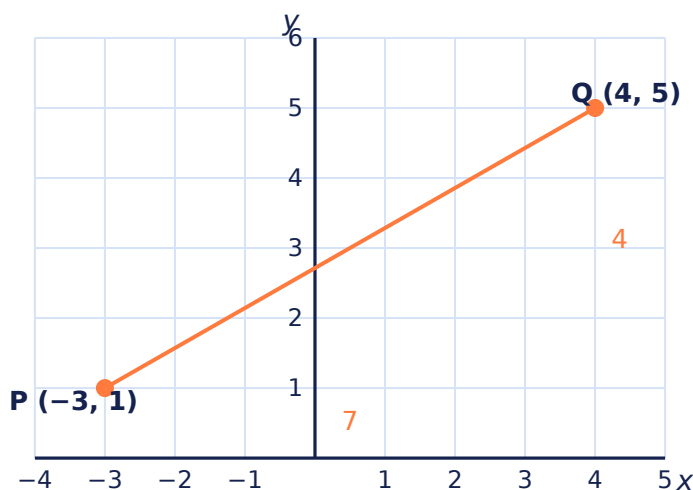
Grade 10 – Geometry math · Lesson · Standards: G-GPE.B.4, G-GPE.B.5, G-GPE.B.6, G-GPE.B.7

Quick answer: Coordinate geometry proves shapes with algebra. The distance formula measures sides, the midpoint formula finds centers of segments, equal slopes show parallel lines, slopes that multiply to -1 show perpendicular lines, and a ratio formula locates a point that divides a segment.

Standards: G-GPE.B.4, G-GPE.B.5, G-GPE.B.6, G-GPE.B.7 · **Level:** Grade 10 Geometry

How do you find the distance between two points?

Any two points on the coordinate plane are the ends of a hypotenuse. Draw a horizontal leg and a vertical leg, and the Pythagorean Theorem does the rest.



Legs of 7 and 4 give $PQ = \sqrt{65}$.

Distance and midpoint

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

What do slopes tell you about two lines?

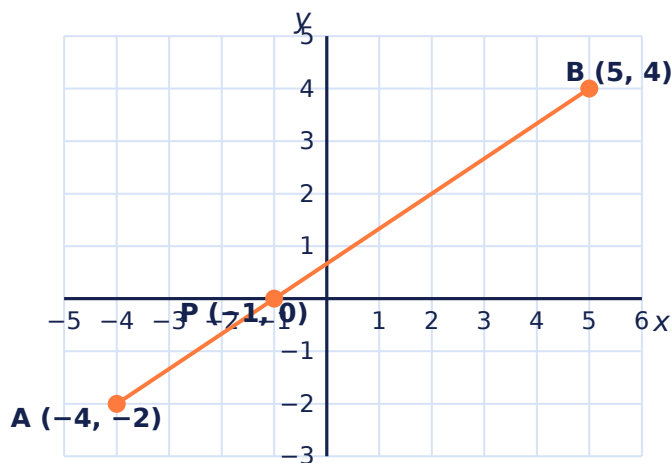
Slope criteria

- Parallel lines have equal slopes.
- Perpendicular lines have slopes whose product is -1 (opposite reciprocals).

How do you find a point that splits a segment in a given ratio?

To locate P on AB with $AP : PB = m : n$, move the fraction $m/(m + n)$ of the way from A to B in each coordinate.

$$P = (x_1 + \frac{m}{m+n}(x_2 - x_1), y_1 + \frac{m}{m+n}(y_2 - y_1))$$



$P(-1, 0)$ splits AB in the ratio $1 : 2$.

How do you prove a figure type with coordinates?

Method

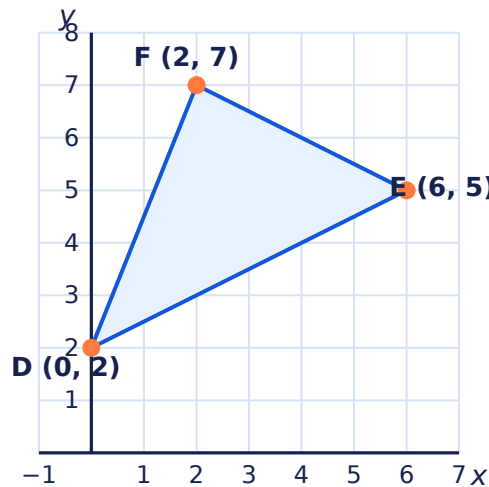
1. Plot the vertices and make a prediction (isosceles, right, parallelogram, rectangle).
2. Choose the test: side lengths for equal sides, slopes for parallel or perpendicular sides, midpoints for diagonals.
3. Compute every quantity the test needs.
4. State the conclusion with the numbers as evidence.

Example

Is the triangle with vertices $D(0, 2)$, $E(6, 5)$ and $F(2, 7)$ a right triangle?

Slope $DE = \frac{3}{6} = \frac{1}{2}$, Slope $DF = \frac{5}{2}$, Slope $EF = \frac{2}{(-4)} = -\frac{1}{2}$.

Slope $DE \cdot \text{slope } EF = \frac{1}{2} \cdot (-\frac{1}{2}) = -\frac{1}{4}$, not -1 , so no side pair is perpendicular. The other pairs give $\frac{5}{2} \cdot \frac{1}{2} = \frac{5}{4}$ and $\frac{5}{2} \cdot (-\frac{1}{2}) = -\frac{5}{4}$. None equals -1 . **Not a right triangle.**



Triangle DEF from the example.

Common mistake

Flipping the slope but forgetting the sign. The perpendicular slope of $\frac{3}{5}$ is $-\frac{5}{3}$, not $\frac{5}{3}$.

Mathbot's tip Keep radicals exact until the end. Comparing squared lengths such as 65 and 65 avoids rounding traps when you test for equal sides.

Key points to remember

- Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- Parallel lines have equal slopes; perpendicular slopes multiply to -1 .
- A point dividing AB in the ratio $m : n$ is the fraction $m/(m + n)$ of the way from A to B.
- Compare squared side lengths to test for isosceles, right or equilateral triangles without decimals.

Proving Shapes with Coordinates and Slopes: frequently asked questions

Why does the distance formula come from the Pythagorean Theorem?

The horizontal change and vertical change between two points form the legs of a right triangle. The segment joining the points is the hypotenuse, so d^2 equals the sum of their squares.

How do I show two lines are perpendicular?

Compute both slopes and multiply. A product of -1 proves the lines are perpendicular. A vertical line and a horizontal line are also perpendicular even though one slope is undefined.

What does a ratio of 2 : 3 mean when locating a point on a segment?

It means the part from the start to the point is 2 units for every 3 units of the part from the point to

the end. The point is $\frac{2}{5}$ of the way from the start.

How can I prove a quadrilateral is a parallelogram on a grid?

Show that both pairs of opposite sides have equal slopes, or that the diagonals share a midpoint. Either result is enough because it proves opposite sides are parallel.

Do I find the area of a polygon with a formula?

For grid figures, enclose the polygon in a rectangle and subtract the corner triangles, or split it into triangles and rectangles. Each piece uses lengths found from the coordinates.