

Proving Shapes with Coordinates and Slopes

Grade 10 – Geometry math · Test Answer Key · Standards: G-GPE.B.4, G-GPE.B.5, G-GPE.B.6, G-GPE.B.7

Each solution shows the formula, the substitution and the conclusion. The scoring notes show how points are earned for the setup, the arithmetic and the proof statement.

Standards: G-GPE.B.4, G-GPE.B.5, G-GPE.B.6, G-GPE.B.7 · **Level:** Grade 10 Geometry

Question 1 Distance and Midpoint (2 points)

$$RS = \sqrt{(9^2 + (-6)^2)} = \sqrt{117} = 3\sqrt{13} \approx 10.82.$$

$$\text{Midpoint} = \left(\frac{(-6 + 3)}{2}, \frac{(2 + -4)}{2} \right) = (-1.5, -1).$$

$3\sqrt{13}$; $(-1.5, -1)$

Scoring: 1 point each.

Question 2 Parallel, Perpendicular or Neither (2 points)

$$\text{Slope } a = \frac{(0 - 4)}{(6 - 0)} = -\frac{2}{3}. \text{ Slope } b = \frac{(-1 - (-5))}{(4 - (-2))} = \frac{4}{6} = \frac{2}{3}. \text{ Slope } c = \frac{3}{2}.$$

$$a \text{ and } b: -\frac{2}{3} \cdot \frac{2}{3} = -\frac{4}{9} \text{ — neither parallel nor perpendicular.}$$

$$a \text{ and } c: -\frac{2}{3} \cdot \frac{3}{2} = -1 \text{ — perpendicular.}$$

a, b : neither; a, c : perpendicular

Scoring: 1 point per pair.

Question 3 Line Through a Point (2 points)

$$\text{The perpendicular slope is } -\frac{3}{4}. \text{ Then } -1 = -\frac{3}{4}(4) + b = -3 + b, \text{ so } b = 2.$$

$$y = -\frac{3}{4}x + 2$$

Scoring: 1 point for the slope, 1 point for the intercept.

Question 4 Splitting a Segment (2 points)

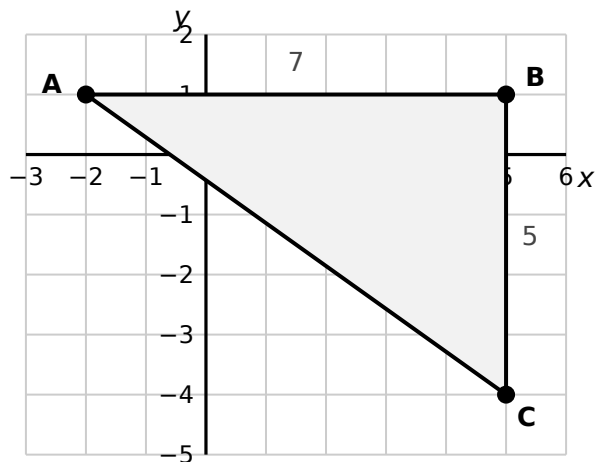
P is $\frac{2}{5}$ of the way from A to B . Changes: 20 in x , -10 in y .

$$x: -6 + \frac{2}{5}(20) = 2. \quad y: 3 + \frac{2}{5}(-10) = -1.$$

$$P = (2, -1)$$

Scoring: 1 point for the setup with the fraction $\frac{2}{5}$, 1 point for the coordinates.

Question 5 Area and Perimeter (3 points)



Legs 7 and 5.

$AB = 7$ and $BC = 5$ with a right angle at B . Area = $\frac{1}{2} \cdot 7 \cdot 5 = 17.5$. $AC = \sqrt{74}$.

Area = 17.5; perimeter = $12 + \sqrt{74}$

Scoring: 1 point for area, 1 point for $\sqrt{74}$, 1 point for the perimeter.

Question 6 Is It Isosceles? (3 points)

$$KL = \sqrt{4^2 + 7^2} = \sqrt{65}. \quad LM = \sqrt{4^2 + (-7)^2} = \sqrt{65}. \quad KM = \sqrt{8^2 + 0^2} = 8.$$

$KL = LM$, so the triangle is isosceles with vertex angle at L .

Isosceles; vertex angle at L

Scoring: 1 point for each of the two equal sides, 1 point for the vertex.

Question 7 Prove a Parallelogram (3 points)

Diagonal EG midpoint = $(\frac{-2+6}{2}, \frac{1+4}{2}) = (2, 2.5)$. Diagonal FH midpoint = $(\frac{3+1}{2}, \frac{0+5}{2}) = (2, 2.5)$.

The diagonals share a midpoint, so $EFGH$ is a parallelogram.

Scoring: 1 point for each midpoint, 1 point for the conclusion.

Question 8 Fourth Vertex (3 points)

$S = P + R - Q = (0 + 9 - 7, 0 + 7 - 2) = (2, 5)$. Check: the diagonals PR and QS both have midpoint $(4.5, 3.5)$.

Enclose the parallelogram in the rectangle from $(0, 0)$ to $(9, 7)$, area 63. The leftover corners are triangles of area 7, 5, 7 and 5, plus two 2-by-2 squares of area 4 each, for a total of 32. So the area is $63 - 32 = 31$.

$S = (2, 5)$; area = 31

Scoring: 1 point for S , 1 point for the check, 1 point for the area.