

Proving Shapes with Coordinates and Slopes

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-GPE.B.4, G-GPE.B.5, G-GPE.B.6, G-GPE.B.7

Each solution shows the coordinates substituted into the formula, keeps radicals exact where possible, and gives a decimal check. Use the slope products and midpoint values to see where a proof went wrong.

Standards: G-GPE.B.4, G-GPE.B.5, G-GPE.B.6, G-GPE.B.7 · **Level:** Grade 10 Geometry

Problem 1 Distance Between Two Points

$$d = \sqrt{((4 - (-3))^2 + (6 - 1)^2)} = \sqrt{7^2 + 5^2} = \sqrt{74}.$$

$$\sqrt{74} \approx 8.60.$$

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Problem 2 Midpoint of a Segment

$$x: (-7 + 5) \div 2 = -1. \quad y: (4 + -10) \div 2 = -3.$$

$$(-1, -3)$$

Problem 3 Missing Endpoint

$$3 = (8 + x) \div 2, \text{ so } x = -2. \quad -2 = (1 + y) \div 2, \text{ so } y = -5.$$

$$K = (-2, -5)$$

Problem 4 Parallel or Not?

$$\text{Slope of } p = \frac{(8 - 2)}{(4 - 1)} = \frac{6}{3} = 2.$$

$$\text{Slope of } q = \frac{(5 - (-3))}{(2 - (-2))} = \frac{8}{4} = 2.$$

Equal slopes, so the lines are parallel.

Problem 5 Perpendicular or Not?

$$\text{Slope of } m = \frac{3}{4}. \quad \text{Slope of } n = \frac{(-8)}{6} = -\frac{4}{3}.$$

$$\text{Product} = \frac{3}{4} \cdot \left(-\frac{4}{3}\right) = -1.$$

Yes, the lines are perpendicular.

Problem 6 Parallel Line Through a Point

Parallel lines share the slope -4 . Use $y = -4x + b$ with $(2, -3)$: $-3 = -8 + b$, so $b = 5$.

$$\mathbf{y = -4x + 5}$$

Problem 7 Perpendicular Line Through a Point

Solve for y : $y = \frac{3}{2}x - 4$, slope $\frac{3}{2}$. The perpendicular slope is $-\frac{2}{3}$.

$$y - 1 = -\frac{2}{3}(x + 6), \text{ so } y = -\frac{2}{3}x - 4 + 1 = -\frac{2}{3}x - 3.$$

$$\mathbf{y = -\frac{2}{3}x - 3}$$

Problem 8 Dividing a Segment 1 : 3

The parts total $1 + 3 = 4$, so the point is $\frac{1}{4}$ of the way from A to B . The changes are 16 in x and -12 in y .

$$x: -6 + \frac{1}{4}(16) = -2. \quad y: 2 + \frac{1}{4}(-12) = -1.$$

$$\mathbf{P = (-2, -1)}$$

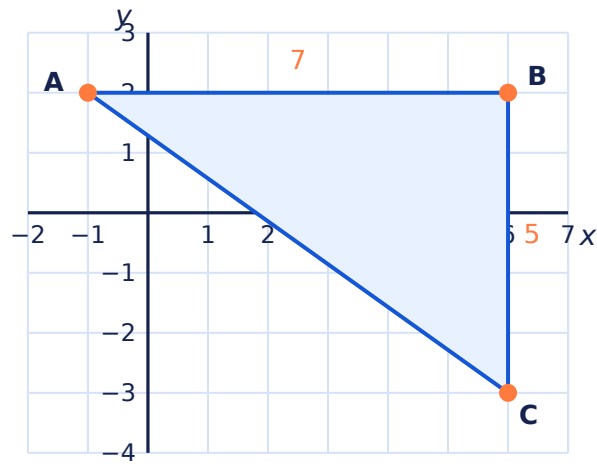
Problem 9 Dividing a Segment 3 : 2

P is $\frac{3}{5}$ of the way from A to B . The changes are 15 in x and -10 in y .

$$x: -5 + \frac{3}{5}(15) = 4. \quad y: 2 + \frac{3}{5}(-10) = -4.$$

$$\mathbf{P = (4, -4)}$$

Problem 10 Perimeter and Area of a Right Triangle



Legs 7 and 5.

$AB = 7$ (horizontal) and $BC = 5$ (vertical), so the triangle has a right angle at B . $AC = \sqrt{7^2 + 5^2} = \sqrt{74} \approx 8.6$.

Perimeter = $12 + \sqrt{74} \approx 20.6$. Area = $\frac{1}{2} \cdot 7 \cdot 5 = 17.5$.

Perimeter = $12 + \sqrt{74} \approx 20.6$; area = 17.5 square units

Problem 11 Area by Enclosing Rectangle

The enclosing rectangle runs from $x = 1$ to 7 and $y = 1$ to 8 : 6 by 7, area 42.

Corner triangles: $\frac{1}{2} \cdot 6 \cdot 2 = 6$ (bottom), $\frac{1}{2} \cdot 3 \cdot 5 = 7.5$ (upper right), $\frac{1}{2} \cdot 3 \cdot 7 = 10.5$ (left).

Area = $42 - 6 - 7.5 - 10.5 = 18$.

18 square units

Problem 12 Classify by Side Lengths

$DE = \sqrt{5^2 + 4^2} = \sqrt{41}$. $EF = \sqrt{5^2 + (-4)^2} = \sqrt{41}$. $DF = \sqrt{10^2 + 0^2} = 10$.

Two sides are equal ($\sqrt{41} \approx 6.40$) and the third is 10.

Isosceles

Problem 13 Prove a Right Triangle

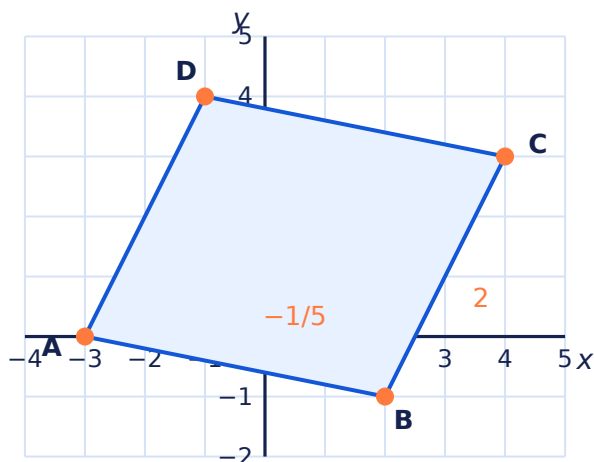
Slope $PQ = \frac{5}{3}$. Slope $PR = \frac{(-6)}{10} = -\frac{3}{5}$.

Product = $\frac{5}{3} \cdot (-\frac{3}{5}) = -1$, so $PQ \perp PR$ and the right angle is at P .

$$PQ = \sqrt{9 + 25} = \sqrt{34}. \quad PR = \sqrt{100 + 36} = \sqrt{136} = 2\sqrt{34}. \quad \text{Area} = \frac{1}{2} \cdot \sqrt{34} \cdot 2\sqrt{34} = 34.$$

Right angle at P; area = 34 square units

Problem 14 Prove a Parallelogram



Opposite sides share slopes $-1/5$ and 2 .

$$\text{Slope } AB = \frac{(-1 - 0)}{(2 - (-3))} = -\frac{1}{5}. \quad \text{Slope } DC = \frac{(3 - 4)}{(4 - (-1))} = -\frac{1}{5}.$$

$$\text{Slope } AD = \frac{4}{2} = 2. \quad \text{Slope } BC = \frac{4}{2} = 2.$$

Both pairs of opposite sides are parallel, so ABCD is a parallelogram.

Problem 15 Prove a Rectangle

Slope $WX = -\frac{2}{5}$, slope $ZY = \frac{(2 - 4)}{(4 - (-1))} = -\frac{2}{5}$. Slope $XY = \frac{5}{2}$, slope $WZ = \frac{5}{2}$. Opposite sides are parallel, so it is a parallelogram.

Slope $WX \cdot \text{slope } XY = -\frac{2}{5} \cdot \frac{5}{2} = -1$, so there is a right angle at X.

Diagonals: $WY = \sqrt{7^2 + 3^2} = \sqrt{58}$ and $XZ = \sqrt{(-3)^2 + 7^2} = \sqrt{58}$, equal as expected for a rectangle.

A parallelogram with a right angle is a rectangle.

Problem 16 Hikers Meet Halfway

$$\text{Midpoint} = \left(\frac{(-4 + 8)}{2}, \frac{(2 + -6)}{2} \right) = (2, -2).$$

Distance from Imani = $\sqrt{6^2 + (-4)^2} = \sqrt{52} \approx 7.21$. Corbin walks the same distance.

Meet at (2, -2); each walks ≈ 7.2 mi

Problem 17 Picnic Table on the Trail

$x: \frac{2}{3}(18) = 12$. $y: \frac{2}{3}(12) = 8$. So the table is at (12, 8).

Distance = $\sqrt{(12^2 + 8^2)} = \sqrt{208} \approx 14.42$ units = 1,442 yards.

(12, 8); about 1,442 yards

Problem 18 Fencing a Trapezoid Field

(a) AB and DC are horizontal, so they are parallel; the other sides have slopes $\frac{5}{2}$ and $-\frac{5}{2}$, not parallel. $AD = \sqrt{(2^2 + 5^2)} = \sqrt{29}$ and $BC = \sqrt{((-2)^2 + 5^2)} = \sqrt{29}$, so the legs are equal: isosceles trapezoid.

(b) Perimeter = $10 + 6 + 2\sqrt{29} = 16 + 2\sqrt{29} \approx 26.77$ units = 268 yards. Area = $\frac{1}{2}(10 + 6)(5) = 40$ square units = 4,000 square yards.

(b) Fence $\approx$ 268 yd; area = 4,000 yd²

Problem 19 Triangular Park

Enclosing rectangle: x from 1 to 9 and y from 2 to 10, so 8 by $8 = 64$.

Corner triangles: $\frac{1}{2} \cdot 8 \cdot 2 = 8$, $\frac{1}{2} \cdot 4 \cdot 6 = 12$, $\frac{1}{2} \cdot 4 \cdot 8 = 16$. Area = $64 - 8 - 12 - 16 = 28$.

Sides: $\sqrt{(64 + 4)} = \sqrt{68} \approx 8.25$, $\sqrt{(16 + 36)} = \sqrt{52} \approx 7.21$, $\sqrt{(16 + 64)} = \sqrt{80} \approx 8.94$.

Area = 28 square blocks; perimeter $\approx$ 24.4 blocks

Problem 20 Midpoints Form a Parallelogram

Midpoints: $AB \rightarrow (4, 1)$; $BC \rightarrow (9, 5.5)$; $CD \rightarrow (6, 8)$; $DA \rightarrow (1, 3.5)$.

Slope (4, 1) to (9, 5.5) = $\frac{4.5}{5} = 0.9$; slope (1, 3.5) to (6, 8) = $\frac{4.5}{5} = 0.9$.

Slope (9, 5.5) to (6, 8) = $\frac{2.5}{(-3)}$; slope (4, 1) to (1, 3.5) = $\frac{2.5}{(-3)}$.

Opposite sides have equal slopes, so the midpoint quadrilateral is a parallelogram.