

Cross Sections, Rotations and Geometric Modeling

Grade 10 – Geometry math · Lesson · Standards: G-GMD.B.4, G-MG.A.1, G-MG.A.2, G-MG.A.3

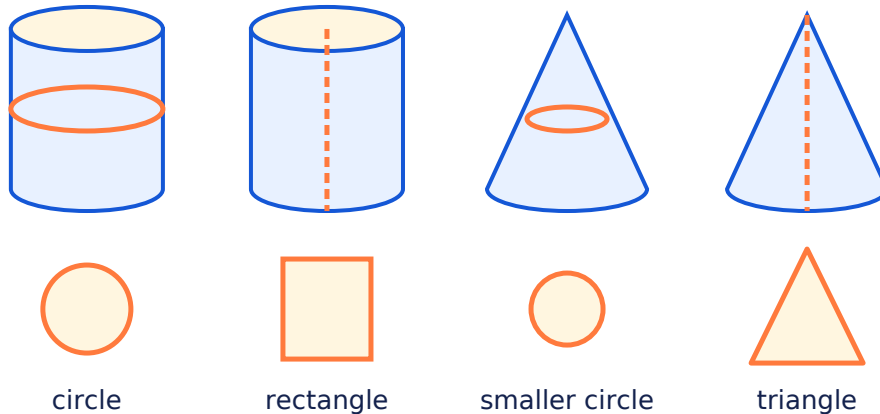
Quick answer: A cross section is the flat shape formed when a plane slices a solid, such as a circle from a cylinder cut parallel to its base. Rotating a rectangle, right triangle or half disk about a line creates a cylinder, cone or sphere. Density compares a quantity to area or volume.

Standards: G-GMD.B.4, G-MG.A.1, G-MG.A.2, G-MG.A.3 · **Level:** Grade 10 Geometry

What is a cross section?

Cross section

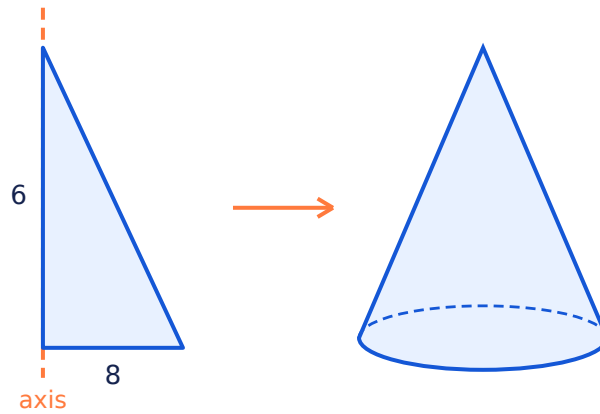
A cross section is the two-dimensional shape you see when a plane slices through a solid. The shape depends on the solid and on the angle and position of the plane.



Cross sections of a cylinder and a cone.

What solid does a rotation make?

Spin a flat figure around a line and it sweeps out a solid. A rectangle turned about one of its sides makes a cylinder, a right triangle turned about a leg makes a cone, and a half disk turned about its diameter makes a sphere.



A right triangle rotated about a leg: the leg on the axis is the height, the other leg is the radius.

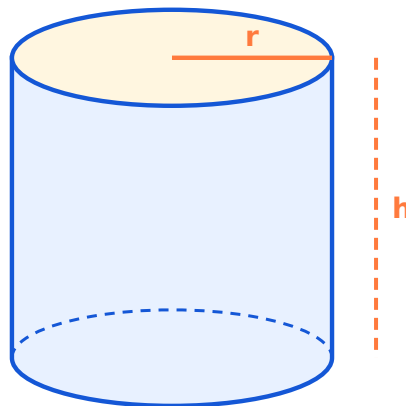
Rotation facts

- The side on the axis becomes the height; the side perpendicular to the axis becomes the radius.
- Any slice perpendicular to the axis of a solid of rotation is a circle.

How do density and modeling work?

Density compares an amount to the space it fills: population per square mile, mass per cubic centimeter or BTUs per cubic foot.

$$\text{Density} = \frac{\text{amount}}{(\text{area or volume})}$$



A soup can modeled as a cylinder: $V = \pi r^2 h$.

Method for modeling problems

1. Choose a simple solid or shape for the object and name what each dimension stands for.
2. Write down every constraint (capacity, cost, size limit).
3. Find the area or volume with the right formula.
4. Apply the density or rate, then check that the answer makes sense in context.

Example

A town of 41,500 people covers 22.5 square miles. Its population density is $41,500 \div 22.5 \approx 1,844$ people per square mile.

Common mistake

Using a diameter as a radius when modeling a round object. Divide a diameter by 2 before squaring.

Mathbot's tip For a similar cut of a cone or pyramid, every length scales by the same ratio. Areas scale by its square and volumes by its cube.

Key points to remember

- A slice parallel to the base of a cylinder or cone is a circle; a slice through the axis of a cylinder is a rectangle.
- A rectangle rotates into a cylinder, a right triangle into a cone, a half disk into a sphere.
- Density = amount $\div$ area (people per square mile) or amount $\div$ volume (BTUs per cubic foot).
- Modeling means choosing a simple shape, applying constraints and checking the result in context.

Cross Sections, Rotations and Geometric Modeling: frequently asked questions

Can a cross section of a cube be a triangle?

Yes. A plane that cuts off a corner of a cube meets three faces and forms a triangle. Planes at other angles can make squares, rectangles, pentagons or hexagons.

Which part of a flat figure becomes the radius when it rotates?

The distance from the axis to the farthest point of the figure becomes the radius. The length along the axis becomes the height.

What is population density?

It is the number of people divided by the land area, for example 3,900 people per square mile. It measures how crowded a region is, not how many people live there.

What does BTU per cubic foot measure?

It is a heating or cooling density: the energy needed per cubic foot of room space. Multiply the room volume by the rate to find the total BTUs required.

How do I choose a shape to model a real object?

Pick the simplest solid whose dimensions you can measure, such as a cylinder for a can or a cone for a funnel, and state your assumptions so the answer can be checked.