

Cross Sections, Rotations and Geometric Modeling

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-GMD.B.4, G-MG.A.1, G-MG.A.2, G-MG.A.3

Each solution states the model first, then the formula and the rounded answer with units. When a result differs, check the scale factor of similar slices and whether density units were matched to area or volume.

Standards: G-GMD.B.4, G-MG.A.1, G-MG.A.2, G-MG.A.3 · **Level:** Grade 10 Geometry

Problem 1 Name the Cross Section

(a) circle. (b) rectangle. (c) circle (smaller than the base). (d) triangle (isosceles). (e) circle (a great circle with the same radius as the sphere).

(a) circle (b) rectangle (c) circle (d) triangle (e) circle

Problem 2 Slices of a Rectangular Box

A slice parallel to a face of a rectangular box is congruent to that face.

(a) $8 \cdot 5 = 40$. (b) $5 \cdot 3 = 15$.

(a) 40 in² (b) 15 in²

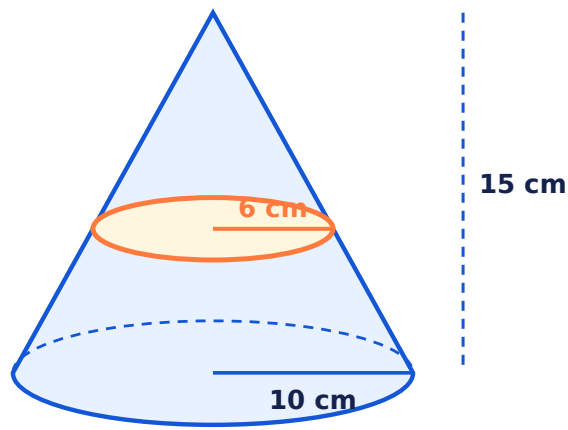
Problem 3 Slicing a Pyramid at Half Height

The cross section is similar to the base with scale factor $\frac{1}{2}$. Its sides are 6 ft and 4 ft.

Area = $6 \cdot 4 = 24$, which is $\frac{1}{4}$ of the base area, 96.

6 ft by 4 ft; area 24 ft²

Problem 4 Slicing a Cone



Cross section radius 6 cm.

The small cone is similar to the large one with scale factor $\frac{9}{15} = \frac{3}{5}$.

Radius = $10 \cdot \frac{3}{5} = 6$. Area = $\pi(6^2) = 36\pi \approx 113.1$.

Radius 6 cm; area $36\pi \text{ cm}^2 \approx 113.1 \text{ cm}^2$

Problem 5 Slicing a Sphere

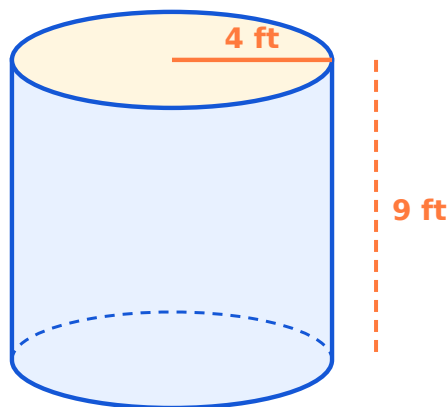
The sphere radius is the hypotenuse of a right triangle whose legs are the distance 8 and the cross-section radius x .

$x = \sqrt{(17^2 - 8^2)} = \sqrt{225} = 15$. Area = $\pi(15^2) = 225\pi \approx 706.9$.

Radius 15 cm; area $225\pi \text{ cm}^2 \approx 706.9 \text{ cm}^2$

Problem 6 Rectangle Rotated into a Cylinder

The solid is a cylinder with radius 4 and height 9.



Cylinder with $r = 4$ and $h = 9$.

$$V = \pi(4^2)(9) = 144\pi \approx 452.4.$$

Cylinder; $144\pi \text{ ft}^3 \approx 452.4 \text{ ft}^3$

Problem 7 Triangle Rotated Two Ways

(a) Radius 6, height 8: $V = \frac{1}{3}\pi(6^2)(8) = 96\pi \approx 301.6.$

(b) Radius 8, height 6: $V = \frac{1}{3}\pi(8^2)(6) = 128\pi \approx 402.1.$

Both solids are cones, but the one with the larger radius holds more.

(a) $96\pi \approx 301.6$ (b) $128\pi \approx 402.1$

Problem 8 Half Disk Rotated into a Sphere

The solid is a sphere of radius 5. $V = \frac{4}{3}\pi(5^3) = \frac{500\pi}{3} \approx 523.6.$

Sphere; $\approx 523.6 \text{ cm}^3$

Problem 9 Isosceles Triangle About Its Altitude

The altitude splits the triangle into two right triangles. The solid is a cone with radius 5 (half the base) and height 12.

$$V = \frac{1}{3}\pi(5^2)(12) = 100\pi \approx 314.16.$$

Cone; $\approx 314.2 \text{ in}^3$

Problem 10 Population Density

$$\text{Density} = 842,600 \div 197 \approx 4277.16.$$

≈ 4277 people per square mile

Problem 11 Mass from Density

$$\text{Volume} = \pi(3^2)(10) = 90\pi \approx 282.74 \text{ cm}^3.$$

$$\text{Mass} = 2.7 \cdot 282.74 \approx 763.41 \text{ g}.$$

$\approx 763 \text{ g}$

Problem 12 Heating a Room

(a) Volume = $18 \cdot 14 \cdot 8 = 2,016 \text{ ft}^3$. BTUs = $2,016 \cdot 22 = 44,352$.

(b) 40,000 is less than 44,352, so the heater is not enough.

(a) 44,352 BTUs (b) No

Problem 13 Does the Soup Can Match Its Label?

The radius is 3.3 cm. $V = \pi(3.3^2)(10.2) \approx 348.96 \text{ cm}^3 \approx 349 \text{ mL}$.

This is within about 1 mL (0.3%) of 350, so the claim is reasonable.

$\approx 349 \text{ mL}$; reasonable

Problem 14 Volume of a Tree Trunk

Radius = $\frac{94}{2\pi} \approx 14.96 \text{ in.}$

Height = 30 ft = 360 in. $V = \pi(14.96)^2(360) \approx 253,133 \text{ in}^3$.

Convert: $\div 1,728 \approx 146.49 \text{ ft}^3$.

Radius $\approx 15 \text{ in}$; volume $\approx 146.5 \text{ ft}^3$

Problem 15 Design a Can

$V = \pi r^2(2r) = 2\pi r^3 = 500$, so $r^3 = \frac{250}{\pi} \approx 79.58$.

$r \approx 4.3 \text{ cm}$ and $h = 2r \approx 8.6 \text{ cm}$.

$r \approx 4.3 \text{ cm}$; $h \approx 8.6 \text{ cm}$

Problem 16 Which Town Is Denser?

Town A: $18,400 \div 7.2 \approx 2555.56$. Town B: $31,500 \div 13.5 \approx 2333.33$.

Difference ≈ 222.22 .

Town A by about 222 people per square mile

Problem 17 Tank Design

Volume needed = $12,000 \div 7.48 \approx 1604.28 \text{ ft}^3$.

$\pi(5^2)h = 1604.28$, so $h = 1604.28 \div (25\pi) \approx 20.43$.

$\approx 20.4 \text{ ft}$

Problem 18 Cans in a Box

Box: $4 \cdot 6.4 = 25.6$ by $3 \cdot 6.4 = 19.2$ by 11 , volume = 5406.72 cm^3 .

Cans: $12 \cdot \pi(3.2^2)(11) \approx 4246.43 \text{ cm}^3$.

Percent = $4246.43 \div 5406.72 \approx 78.54\%$.

$\approx 78.5\%$

Problem 19 Water in a Conical Cup

The scale factor is $\frac{8}{12} = \frac{2}{3}$. The surface radius is $4.5 \cdot \frac{2}{3} = 3 \text{ cm}$.

Water volume = $\frac{1}{3}\pi(3^2)(8) = 24\pi \approx 75.4 \text{ cm}^3$.

Cup volume = $\frac{1}{3}\pi(4.5^2)(12) = 81\pi$. Fraction = $24\pi \div 81\pi = \frac{8}{27} \approx 29.6\%$.

Radius 3 cm; $\approx 75.4 \text{ cm}^3$; $8/27$ of the cup (about 29.6%)

Problem 20 Shipping Tube Box

Box = $4.5 \cdot 4.5 \cdot 36 = 729 \text{ in}^3$. Tube = $\pi(2.25^2)(36) \approx 572.56 \text{ in}^3$.

Empty = $729 - 572.56 = 156.44 \text{ in}^3$, which is 21.46% of the box.

Box 729 in^3 ; tube $\approx 572.6 \text{ in}^3$; empty $\approx 21.5\%$