

Dilations, Scale Factor and Similar Figures

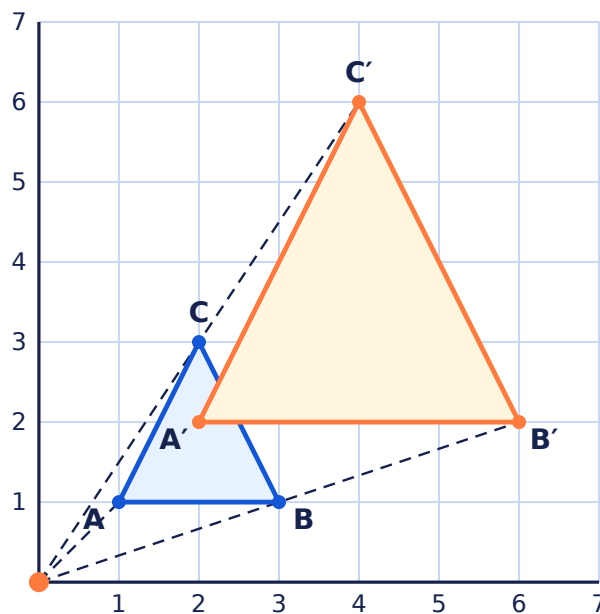
Grade 10 – Geometry math · Lesson · Standards: G-SRT.A.1, G-SRT.A.2, G-SRT.A.3

Quick answer: A dilation scales a figure from a center point by a scale factor k , multiplying every distance from the center by k . It keeps angles equal and maps lines to parallel lines, except lines through the center, which stay put. Figures are similar when dilations and rigid motions map one onto the other. AA proves triangles similar.

Standards: G-SRT.A.1, G-SRT.A.2, G-SRT.A.3 · **Level:** Grade 10 Geometry

What is a dilation?

A **dilation** is a transformation with a fixed **center** C and a **scale factor** $k > 0$. Each point P moves to P' on ray CP so that $CP' = k \cdot CP$. If $k > 1$ the figure is enlarged; if $0 < k < 1$ it is reduced. A dilation is not a rigid motion, because distances change.



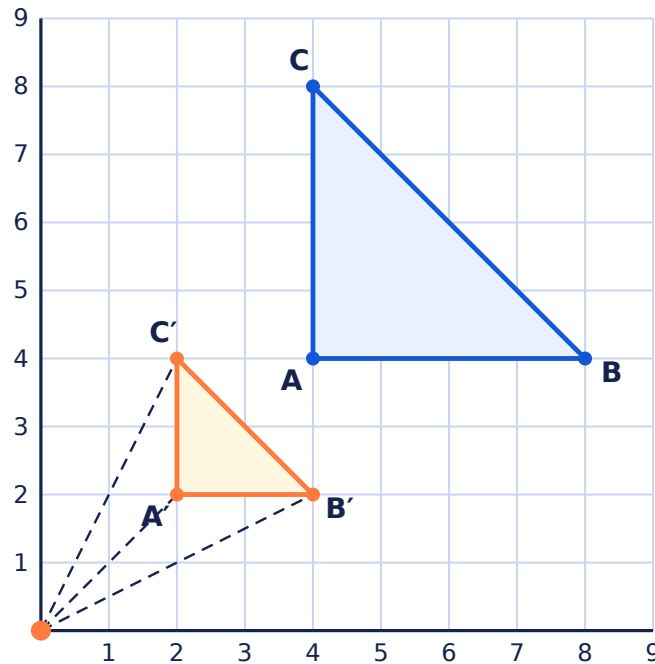
Dilation about the origin with $k = 2$: $(x, y) \rightarrow (2x, 2y)$. Dashed rays pass through the center.

Coordinate rule

About the origin: $(x, y) \rightarrow (kx, ky)$. About a center (a, b) : $(x, y) \rightarrow (a + k(x - a), b + k(y - b))$.

Which properties does a dilation have?

Property	Result
Segment lengths	Multiplied by k
Angle measures	Unchanged
A line through the center	Maps onto itself
A line not through the center	Maps to a parallel line
Perimeter and area	Perimeter $\times k$, area $\times k^2$



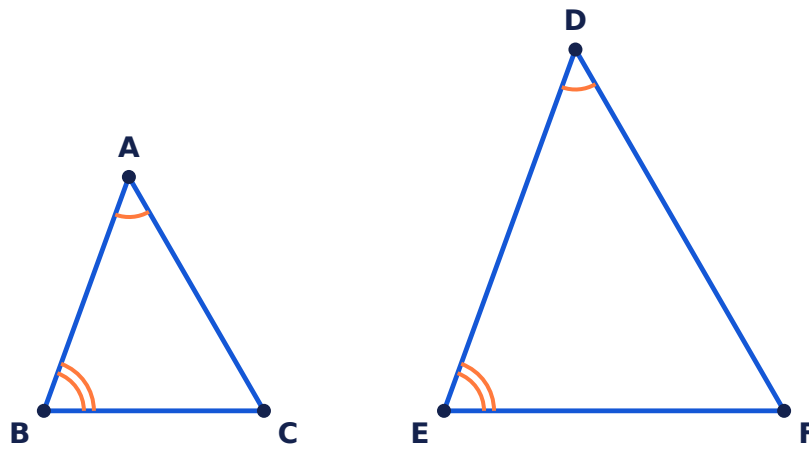
Scale factor $\frac{1}{2}$ shrinks every distance from the center by half.

How do dilations define similarity?

Two figures are **similar** if a sequence of rigid motions and dilations maps one onto the other. Then corresponding angles are congruent and corresponding sides are proportional, and the common ratio is the scale factor. We write $\triangle ABC \sim \triangle DEF$ with the vertices in matching order.

How do you show that triangles are similar?

The **AA criterion** says that if two angles of one triangle are congruent to two angles of another, the triangles are similar. The third angles must match too, since all triangles have angle sum 180° . Dilate one triangle until a side matches, and the triangles become congruent by ASA.



Two pairs of congruent angles make the triangles similar (AA).

Method

1. Find the scale factor by dividing an image length by the matching original length.
2. Check that the same ratio works for another pair of lengths.
3. Multiply or divide other lengths by the scale factor.
4. For coordinates, apply the dilation rule to each vertex.

Worked example

Dilate $P(5, 4)$ about the center $(2, 1)$ with $k = 2$. The vector from the center to P is $(3, 3)$. Doubling gives $(6, 6)$, so $P' = (2 + 6, 1 + 6) = \mathbf{(8, 7)}$.

Common mistake

The scale factor scales *lengths*, so the area changes by k^2 , not k . Doubling a side makes the area four times as large.

Mathbot's tip

Image over original always gives the scale factor: $k = \text{image length} \div \text{original length}$. If it is less than 1, the picture shrank.

Key points to remember

- A dilation with scale factor k multiplies every distance from the center and every length by k .
- Angle measures do not change under a dilation.
- A line through the center stays the same; any other line maps to a parallel line.
- Similar figures have equal corresponding angles and proportional corresponding sides.
- Two pairs of congruent angles prove triangles similar (AA).

Dilations, Scale Factor and Similar Figures: frequently asked questions

What is the difference between a dilation and a rigid motion?

A rigid motion preserves distances, while a dilation multiplies them by the scale factor. A dilation with scale factor 1 is the only one that is also rigid.

How do you find a scale factor?

Divide a length in the image by the matching length in the original. If a 6-inch segment becomes 15 inches, the scale factor is $15 \div 6 = 2.5$.

Why does area change by the square of the scale factor?

Area has two dimensions, and each dimension is multiplied by k . A rectangle with sides tripled has $3 \times 3 = 9$ times the area.

Why do two pairs of equal angles prove triangles similar?

The third angles are forced to be equal because the angles sum to 180° . Dilating one triangle until a side matches then gives congruent triangles by ASA.

What happens to a line that passes through the center of dilation?

It maps onto itself, because every image point stays on the ray from the center. Only the positions of the points along the line change.