

Dilations, Scale Factor and Similar Figures

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-SRT.A.1, G-SRT.A.2, G-SRT.A.3

Each answer applies the dilation rule, finds the scale factor as image over original, and checks the result with a second length or coordinate. The proof lists a reason beside each step.

Standards: G-SRT.A.1, G-SRT.A.2, G-SRT.A.3 · **Level:** Grade 10 Geometry

Problem 1 Dilate a point

Multiply each coordinate by 3: $(3 \cdot 2, 3 \cdot 3) = (6, 9)$.

A'(6, 9)

Problem 2 Shrink a point

Multiply each coordinate by $\frac{1}{2}$: $(4, -3)$.

B'(4, -3)

Problem 3 Find the scale factor

Divide each image coordinate by the original: $10 \div 4 = 2.5$ and $15 \div 6 = 2.5$.

k = 2.5

Problem 4 Length under a dilation

Lengths are multiplied by the scale factor: (a) $7 \times 4 = 28$ and (b) $7 \times \frac{1}{2} = 3.5$.

(a) 28 cm, (b) 3.5 cm

Problem 5 Distances from the center

$CP' = 2.5 \times 6 = 15$. For the second dilation, $k = CP'' \div CP = 9 \div 6 = 1.5$.

CP' = 15; k = 1.5

Problem 6 Dilate a triangle

Multiply each coordinate by 2: $A'(2, 2)$, $B'(8, 2)$, $C'(8, 6)$.

Side lengths: $AB = 3$, $BC = 2$, $AC = \sqrt{9 + 4} = \sqrt{13}$. The image sides are 6, 4 and $2\sqrt{13}$, each twice as long, so the perimeter doubles from $5 + \sqrt{13}$ to $10 + 2\sqrt{13}$.

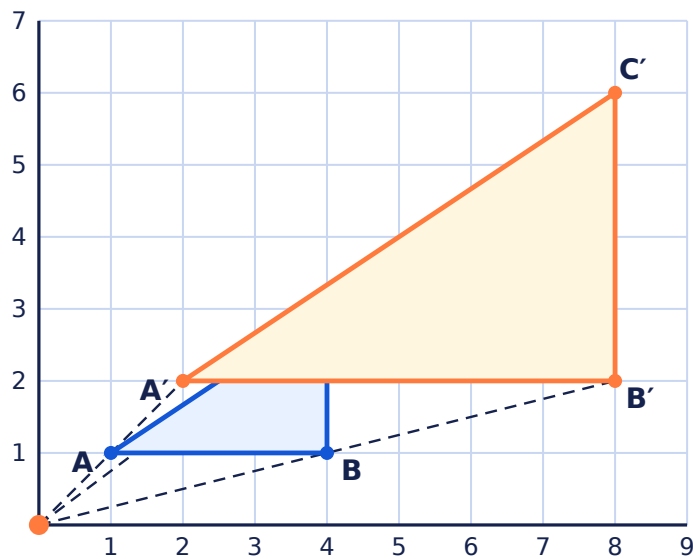
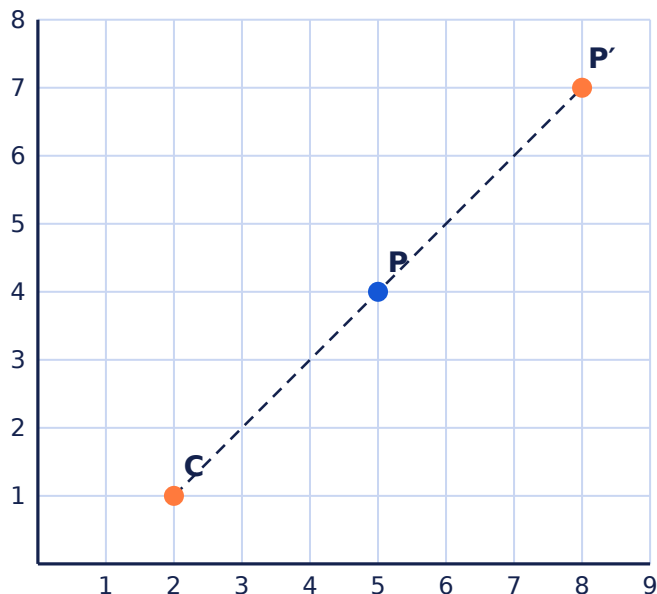


Image with $k = 2$.

$A'(2, 2)$, $B'(8, 2)$, $C'(8, 6)$; perimeter $\times 2$

Problem 7 Dilation about another center



Solved figure: $P'(8, 7)$.

The vector from C to P is $(5 - 2, 4 - 1) = (3, 3)$. Doubling gives $(6, 6)$. Add to the center: $(2 + 6,$

$$1 + 6 = (8, 7).$$

Check: C , P and P' lie on the line $y = x - 1$.

$P'(8, 7)$

Problem 8 A line not through the center

$(0, 3) \rightarrow (0, 9)$ and $(1, 5) \rightarrow (3, 15)$. The slope of the image line is $(15 - 9) \div (3 - 0) = 2$, and its y -intercept is 9, so it is $y = 2x + 9$.

The slope is unchanged, so the image line is parallel to the original.

$y = 2x + 9$, parallel to $y = 2x + 3$

Problem 9 A line through the center

$(1, 3) \rightarrow (4, 12)$. Check the equation: $3(4) = 12$, so $(4, 12)$ lies on $y = 3x$. A line through the center maps onto itself.

$(4, 12)$ is on $y = 3x$

Problem 10 Perimeter and area

The perimeter is multiplied by 4: $20 \times 4 = 80$. The area is multiplied by $4^2 = 16$: $15 \times 16 = 240$.

Perimeter 80; area 240

Problem 11 Write the similarity statement

The third angles are $m\angle C = 180^\circ - 52^\circ - 61^\circ = 67^\circ$ and $m\angle F = 180^\circ - 52^\circ - 67^\circ = 61^\circ$. Both triangles have angles 52° , 61° and 67° , so they are similar by AA.

Matching: $A \leftrightarrow D$ (52°), $B \leftrightarrow F$ (61°), $C \leftrightarrow E$ (67°).

Yes: $\triangle ABC \sim \triangle DFE$

Problem 12 Proportional sides

The scale factor is $DE \div AB = 10 \div 6 = 5/3$. Then $EF = (5/3)(9) = 15$.

$k = 5/3$; $EF = 15$

Problem 13 Is it a dilation

$(0, 0) \rightarrow (1, 0)$, but a dilation about the origin keeps the origin fixed. So it is not a dilation about the origin.

The rule doubles every coordinate and then shifts right by 1, so it is a dilation about the origin with $k = 2$ followed by a translation by $\langle 1, 0 \rangle$. This is a similarity transformation.

Not a dilation about the origin; a dilation ($k = 2$) followed by a translation

Problem 14 A similarity sequence

The length $AB = 2$ and $A'B' = 4$, so $k = 2$. Dilate about the origin: $(0, 0)$, $(4, 0)$, $(0, 6)$.

Then translate by $\langle 4, 1 \rangle$: $(4, 1)$, $(8, 1)$, $(4, 7)$. These match A' , B' , C' . Check BC : $\sqrt{13}$ and $B'C' = \sqrt{(16 + 36)} = 2\sqrt{13}$.

Dilation with $k = 2$ about the origin, then translation by $\langle 4, 1 \rangle$

Problem 15 Scale model of a statue

The statue is 450 centimeters tall, so $k = 18 \div 450 = 0.04$ (or $1/25$).

Model base = $120 \times 0.04 = 4.8$ centimeters. Areas scale by $k^2 = (1/25)^2 = 1/625$, so the model's surface area is 625 times smaller.

$k = 0.04$; base 4.8 cm; area $1/625$ of the statue's

Problem 16 Projector enlargement

The screen image is $3.5 \times 18 = 63$ centimeters by $2.4 \times 18 = 43.2$ centimeters. Its area is $63 \times 43.2 = 2,721.6$ square centimeters.

The slide's area is $3.5 \times 2.4 = 8.4$, so the area grew by $2,721.6 \div 8.4 = 324 = 18^2$.

63 cm by 43.2 cm; area 2,721.6 cm²; area $\times 324$

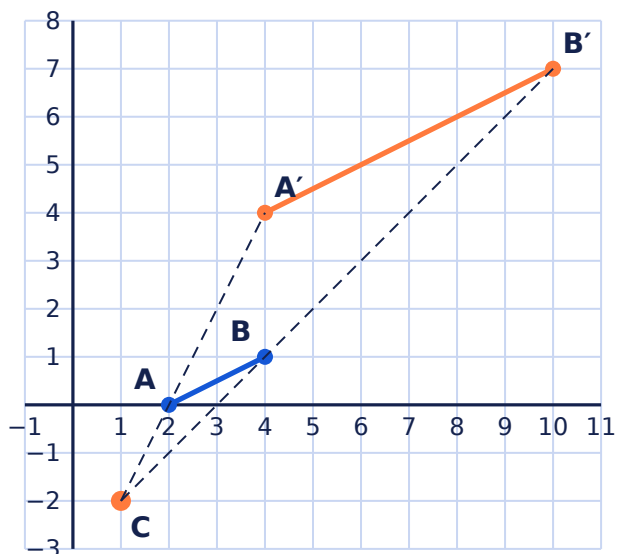
Problem 17 Resizing a photo

The scale factor is $5 \div 8 = 5/8$. The new height is $10 \times 5/8 = 6.25$ inches.

The original perimeter is $2(8 + 10) = 36$ inches, so the new perimeter is $36 \times 5/8 = 22.5$ inches. Check: $2(5 + 6.25) = 22.5$.

$k = 5/8$; height 6.25 in; perimeter 22.5 in

Problem 18 Dilation with a center off the origin



Solved figure: $A'(4, 4)$, $B'(10, 7)$.

A: vector from the center is $(1, 2)$; tripled it is $(3, 6)$; $A' = (1 + 3, -2 + 6) = (4, 4)$.

B: vector $(3, 3)$; tripled $(9, 9)$; $B' = (1 + 9, -2 + 9) = (10, 7)$.

$AB = \sqrt{2^2 + 1^2} = \sqrt{5}$ and $A'B' = \sqrt{6^2 + 3^2} = \sqrt{45} = 3\sqrt{5}$, which is 3 times AB .

$A'(4, 4)$, $B'(10, 7)$; $A'B' = 3 \cdot AB$

Problem 19 Prove the AA similarity

Statement	Reason
1. $\angle A \cong \angle D$ and $\angle B \cong \angle E$	Given
2. Let $k = DE \div AB$. Dilate $\triangle ABC$ about A by k to get $\triangle AB'C'$	Definition of dilation
3. $AB' = k \cdot AB = DE$	Dilation multiplies lengths by k
4. $\angle A \cong \angle A$, $\angle B' \cong \angle B \cong \angle E$	Dilations preserve angle measure; Given
5. $\triangle AB'C' \cong \triangle DEF$	ASA ($\angle A \cong \angle D$, $AB' = DE$, $\angle B' \cong \angle E$)
6. A rigid motion maps $\triangle AB'C'$ onto $\triangle DEF$	Congruent triangles are related by rigid motions
7. $\triangle ABC \sim \triangle DEF$	A dilation followed by a rigid motion maps one onto the other

$\triangle ABC \sim \triangle DEF$

Problem 20 Shadow measurement

The sun's rays are parallel, so each person or tree and its shadow form a right triangle with the same angle at the ground. Each also has a right angle at the base. Two pairs of congruent angles make the triangles similar by AA.

Scale factor from the person's triangle to the tree's triangle: $12 \div 2.55$. The tree is taller by the same factor, or by ratio: $\text{height} \div 12 = 1.7 \div 2.55 = 2/3$, so $\text{height} = (2/3)(12) = 8$.

The tree is 8 meters tall