

Writing and Graphing Circle Equations

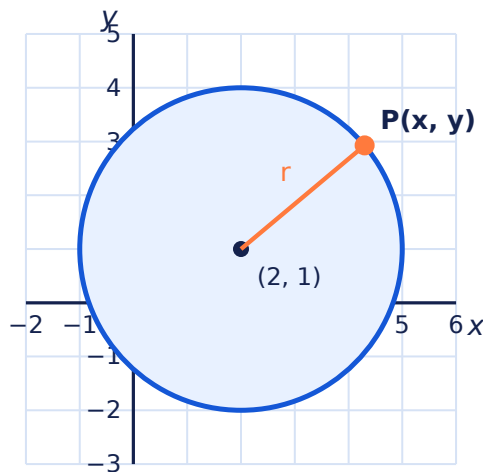
Grade 10 – Geometry math · Lesson · Standards: G-GPE.A.1

Quick answer: The equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$. It comes from the Pythagorean Theorem applied to the horizontal and vertical distances from the center. Completing the square converts any general circle equation into this form.

Standards: G-GPE.A.1 · **Level:** Grade 10 Geometry

Where does the equation of a circle come from?

Think of a circle as every point that stays exactly r units away from a fixed spot (h, k) . Pick a point (x, y) on it. Walking from the center to that point takes a horizontal step of $|x - h|$ and a vertical step of $|y - k|$, and the radius closes a right triangle.



Pythagorean Theorem: $(x - h)^2 + (y - k)^2 = r^2$.

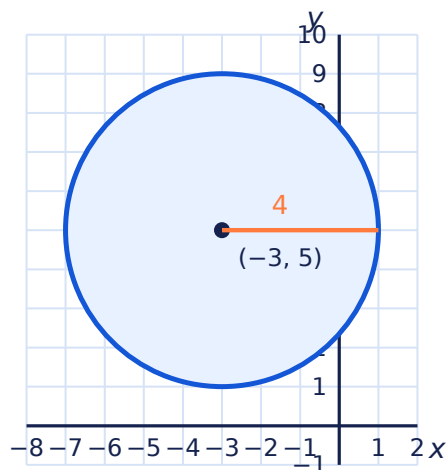
Standard form

$$(x - h)^2 + (y - k)^2 = r^2$$

The center is (h, k) and the radius is r . A circle centered at the origin is $x^2 + y^2 = r^2$.

How do you read the center and radius?

Signs flip: $(x + 3)$ hides $h = -3$, while $(y - 5)$ hides $k = 5$. The constant on the right side equals r^2 , so a square root is still needed.



$(x + 3)^2 + (y - 5)^2 = 16$ has center $(-3, 5)$ and radius 4.

How do you rewrite a general equation by completing the square?

Method

1. Collect the x -terms together, the y -terms together, and send the constant across the equals sign.
2. Square half of each linear coefficient and add both results to each side.
3. Turn each trinomial into a squared binomial.
4. Read off the center and radius, confirming the right side is positive.

Example

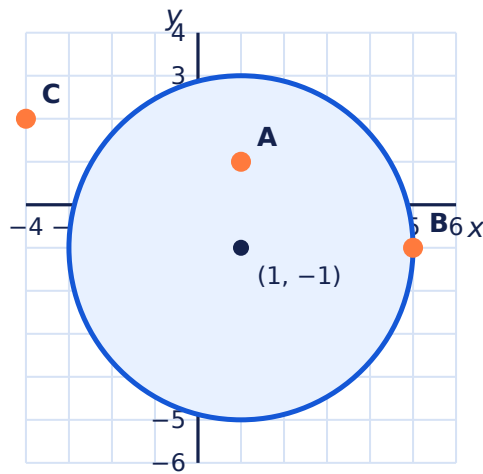
Rewrite $x^2 + y^2 - 10x + 4y - 7 = 0$.

$(x^2 - 10x) + (y^2 + 4y) = 7$. Add 25 and 4: $(x - 5)^2 + (y + 2)^2 = 7 + 25 + 4 = 36$.

Center $(5, -2)$, radius 6.

How do you tell whether a point is inside, on or outside?

Compute $(x - h)^2 + (y - k)^2$ and compare with r^2	Position
less than r^2	inside
equal to r^2	on the circle
greater than r^2	outside



A is inside, B is on the circle, C is outside.

Common mistake

Using the wrong sign for the center, or forgetting to take the square root of the right side to get the radius.

Mathbot's tip After completing the square, check that the right side is positive. If it is zero you have a single point; if it is negative there is no graph.

Key points to remember

- Standard form: $(x - h)^2 + (y - k)^2 = r^2$, center (h, k) , radius r .
- A circle centered at the origin is $x^2 + y^2 = r^2$.
- Complete the square in x and in y to read the center and radius from a general equation.
- A point is inside, on or outside the circle when its squared distance is less than, equal to or greater than r^2 .

Writing and Graphing Circle Equations: frequently asked questions

Why does the circle equation use squares?

Every point on the circle is a distance r from the center, and distance comes from the Pythagorean Theorem. Squaring the horizontal and vertical differences avoids square roots in the equation.

How do I find the center from $(x + 4)^2 + (y - 7)^2 = 9$?

Reverse each sign. The center is $(-4, 7)$, because $x + 4$ equals $x - (-4)$. The radius is $\sqrt{9} = 3$.

What if the squared terms have a coefficient larger than 1?

Divide the whole equation by that coefficient first so that x^2 and y^2 each have coefficient 1, then complete the square.

Can a circle equation have no graph?

Yes. After completing the square, a negative number on the right means no real points satisfy the equation, and zero means the graph is a single point.

How do I write the equation from a diameter?

Find the midpoint of the diameter for the center, then use the distance from the center to an endpoint for the radius.