

Compass and Straightedge Constructions

Grade 10 – Geometry math · Lesson · Standards: G-CO.D.12, G-CO.D.13

Quick answer: A geometric construction uses only a compass and an unmarked straightedge. Equal arcs let you copy segments and angles, bisect segments and angles, and build perpendicular and parallel lines. The radius of a circle steps around it exactly six times, giving an inscribed regular hexagon, and every second vertex gives an equilateral triangle. Each step is justified by congruent triangles.

Standards: G-CO.D.12, G-CO.D.13 · **Level:** Grade 10 Geometry

What tools do geometric constructions allow?

A **construction** uses only a **compass**, which draws circles and arcs, and an unmarked **straightedge**, which draws lines. No ruler marks and no protractor are used. Each step is justified by a definition or theorem, so the result is exact.

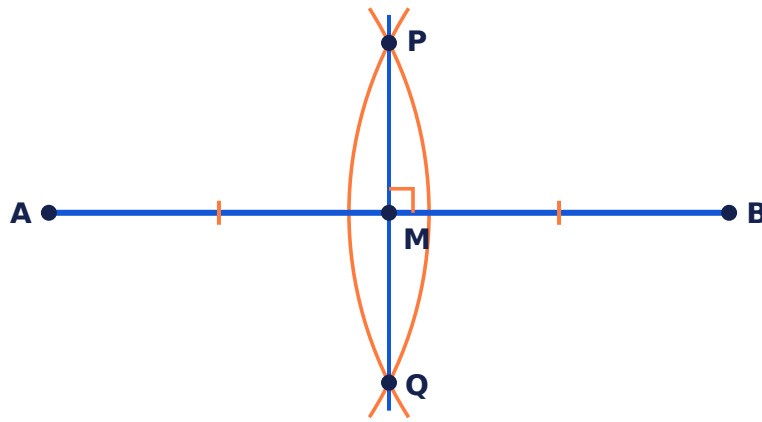
How do you copy a segment or an angle?

Copying

1. **Segment:** draw a ray, set the compass to the length of the original, and mark that distance from the endpoint of the ray.
2. **Angle:** draw an arc across both sides of the original angle. Draw the same arc on a new ray. Copy the chord between the two crossing points onto the new arc, then draw the second side through the new mark.

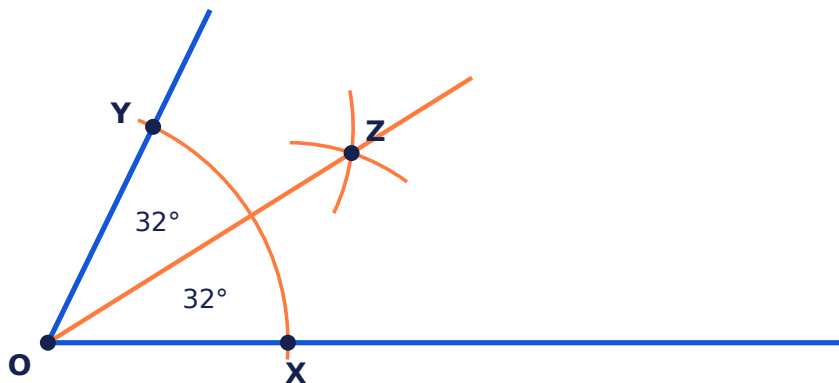
How do you bisect a segment and an angle?

To bisect a segment, open the compass wider than half its length and draw arcs from both endpoints. The arcs meet at two points, and the line through them is the **perpendicular bisector**.



Perpendicular bisector: equal arcs from A and B meet at P and Q; line PQ bisects AB at M.

To bisect an angle, draw an arc centered at the vertex that crosses both sides, then draw equal arcs from those two crossing points. The ray from the vertex through where those arcs meet is the angle bisector.

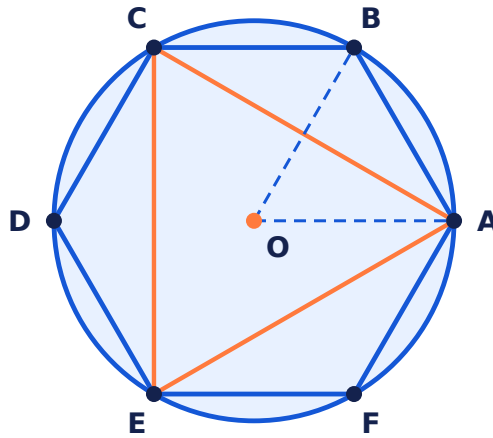


Angle bisector: ray OZ splits a 64° angle into two 32° angles.

Perpendicular and parallel lines come from these tools too. Mark equal distances on both sides of a point to build a perpendicular through it. A parallel through a point is built by copying a corresponding angle, and the converse of the Corresponding Angles Theorem proves the lines parallel.

How do you inscribe regular polygons in a circle?

The radius of a circle fits around its circle exactly six times. Mark the radius around the circle to get a regular **hexagon**. Joining every second vertex gives an inscribed **equilateral triangle**. Drawing two perpendicular diameters gives the vertices of an inscribed **square**.



Hexagon: step the radius around the circle six times. Connect every second vertex for an equilateral triangle.

Shape in a circle of radius r	Side length
Regular hexagon	r
Equilateral triangle	$r\sqrt{3}$
Square	$r\sqrt{2}$

Why the hexagon works

If $OA = OB = r$ and the compass is set to r , then $AB = r$ too. So $\triangle OAB$ is equilateral and $\angle AOB = 60^\circ$. Six such angles make **360°**.

Common mistake

If the compass width changes between the two arcs of a bisector, the construction fails. Keep one setting for matching arcs, and keep it larger than half the segment.

Mathbot's tip

After every construction ask: which two lengths are equal because they are radii of the same arc? Those equal lengths are the proof.

Key points to remember

- Constructions use only a compass and a straightedge with no marks.
- The perpendicular bisector needs a compass opening larger than half the segment.
- An angle bisector arc construction works because of SSS congruent triangles.
- A circle's radius fits around it six times: the inscribed hexagon has side r .
- Every second vertex of the hexagon gives an inscribed equilateral triangle of side $r\sqrt{3}$.

Compass and Straightedge Constructions: frequently asked questions

Why can a compass and straightedge construction be called exact?

Every point is a crossing of lines or circles, and each step is backed by a theorem such as SSS congruence. No measurement is used, so no measuring error enters.

Why must the compass opening be greater than half the segment?

If the radius is half the segment or less, the arcs from the two endpoints touch at most at the midpoint and never cross on both sides. The perpendicular bisector then cannot be found.

How do you construct a parallel line through a point?

Draw a line through the point that crosses the given line, then copy the corresponding angle at the point. The new line is parallel by the converse of the Corresponding Angles Theorem.

Why is the side of an inscribed regular hexagon equal to the radius?

The center and two neighboring vertices form a triangle with two radii and a side. The compass keeps the side equal to the radius, so the triangle is equilateral with a 60° central angle.

How do you inscribe a square in a circle?

Draw a diameter, build its perpendicular bisector to get a second diameter, and join the four points where the diameters meet the circle.