

Compass and Straightedge Constructions

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-CO.D.12, G-CO.D.13

Each answer states what the construction produces, which lengths are equal because they are radii, and how congruent triangles or a theorem justifies the result. Numerical answers include checks.

Standards: G-CO.D.12, G-CO.D.13 · **Level:** Grade 10 Geometry

Problem 1 Order the steps to copy a segment

The correct order is: draw the ray with endpoint C (step 2), set the compass to AB (step 4), draw the arc from C (step 1), and label D (step 3).

2, 4, 1, 3

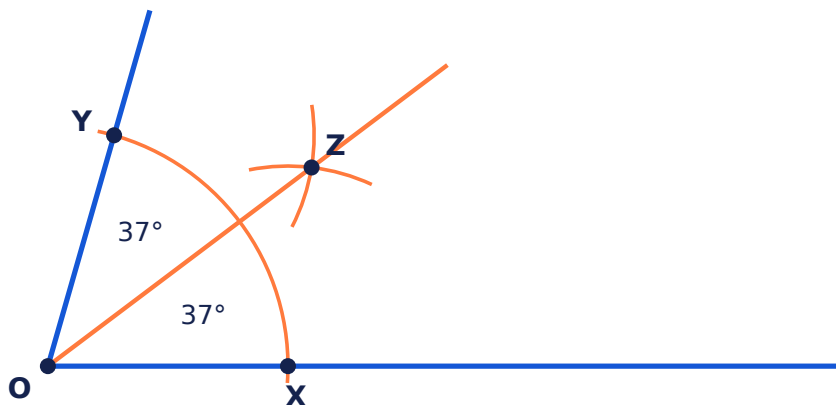
Problem 2 Perpendicular bisector of a segment

The bisector meets AB at its midpoint, so $AM = MB = 12 \div 2 = 6$ centimeters. The bisector is perpendicular to AB , so $\angle AMP = 90^\circ$.

The compass opening must be greater than half of AB , that is, more than 6 centimeters, so that the arcs cross.

$AM = MB = 6$ cm; $\angle AMP = 90^\circ$; opening > 6 cm

Problem 3 Bisect an angle



Solved figure: two angles of 37° .

The bisector splits the angle into two congruent angles, each $74^\circ \div 2 = 37^\circ$.

$$\angle XOZ = \angle ZOY = 37^\circ$$

Problem 4 Copy an angle

A copied angle is congruent to the original, so $\angle EDF$ measures 48° .

The arc radii must be equal (the arc from B and the arc from D), and the chord between the crossing points on the original must equal the chord marked on the new arc. Then the two small triangles are congruent by SSS.

48° ; the arc radii and the chords must match

Problem 5 Equilateral triangle on a segment

AC and BC are radii of arcs with radius AB , so $AC = BC = AB = 6.5$. The triangle is equilateral.

Perimeter $= 3(6.5) = 19.5$ centimeters, and each angle is $180^\circ \div 3 = 60^\circ$.

Perimeter 19.5 cm; all angles 60°

Problem 6 Hexagon in a circle

Each side equals the radius: 5 centimeters. Perimeter $= 6 \cdot 5 = 30$ centimeters. Each central angle is $360^\circ \div 6 = 60^\circ$.

Side 5 cm; perimeter 30 cm; central angle 60°

Problem 7 Why the hexagon construction works

Let O be the center, and let A and B be neighboring marks. OA and OB are radii, and AB equals the compass setting, which is also the radius. All three sides are equal, so $\triangle OAB$ is equilateral and $\angle AOB = 60^\circ$. Six such angles use $6 \times 60^\circ = 360^\circ$, so the marks close the circle exactly.

Equilateral triangles; each central angle is 60°

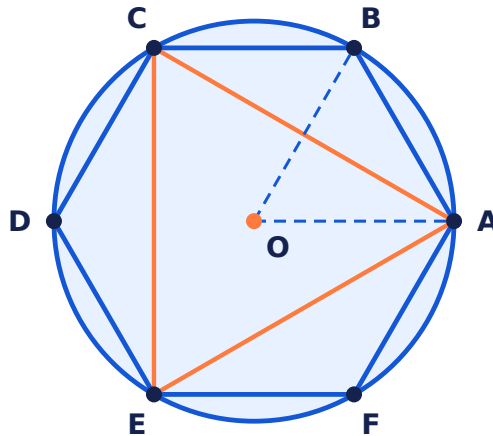
Problem 8 Square in a circle

The diagonals are diameters, so the diagonal is $2(8) = 16$. A square with diagonal d has side $d \div \sqrt{2} = 16 \div \sqrt{2} = 8\sqrt{2} \approx 11.31$.

Area $= (8\sqrt{2})^2 = 128$. Check: the area also equals $\frac{1}{2}d^2 = \frac{1}{2}(256) = 128$.

Diagonal 16; side ≈ 11.31 ; area 128

Problem 9 Triangle in a circle



The triangle joins every second vertex.

The side of the triangle is $r\sqrt{3} = 6\sqrt{3} \approx 10.39$. (It is the long diagonal across two hexagon sides, with a central angle of 120° .)

$$\text{Area} = (\sqrt{3} \div 4)(6\sqrt{3})^2 = (\sqrt{3} \div 4)(108) = 27\sqrt{3} \approx 46.77.$$

Side ≈ 10.39 ; area ≈ 46.77

Problem 10 Parallel line through a point

The copied angle and the original are corresponding angles formed by the transversal through P , and they are congruent. By the converse of the Corresponding Angles Theorem, $n \parallel m$.

Alternate interior angles of parallel lines are congruent, so the alternate interior angle also measures 65° .

Converse of the Corresponding Angles Theorem; 65°

Problem 11 Perpendicular through a point on a line

$PX = PY$ because they are radii of the same arc, so P is the midpoint of XY . The perpendicular bisector of XY passes through the midpoint P and is perpendicular to XY , which lies along l .

It is the perpendicular bisector of XY , so it contains P and meets l at 90°

Problem 12 Perpendicular from a point off the line

$PX = PY$ (same arc) and $QX = QY$ (equal arcs). So both P and Q are equidistant from X and Y , which places both on the perpendicular bisector of XY (converse of the Perpendicular Bisector Theorem). The line through P and Q is therefore perpendicular to XY , that is, to l .

P and Q are both on the perpendicular bisector of XY

Problem 13 Spot the mistake

The opening must exceed half of AB , which is 5 centimeters. With a 4-centimeter opening, the arcs from A and B reach only 4 centimeters across, leaving a gap of 2 centimeters in the middle, so they never cross.

The smallest whole number that works is 6 centimeters.

The arcs do not meet; 6 cm

Problem 14 Perpendicular bisector on a grid

Midpoint: $((-4 + 6) \div 2, (1 + 5) \div 2) = (1, 3)$. Slope of $AB = (5 - 1) \div (6 + 4) = 2/5$, so the perpendicular slope is $-5/2$.

Line: $y - 3 = -(5/2)(x - 1)$, which gives $y = -2.5x + 5.5$. At $x = 3$, $y = -7.5 + 5.5 = -2$, so P lies on it.

$PA^2 = (3 + 4)^2 + (-2 - 1)^2 = 49 + 9 = 58$ and $PB^2 = (3 - 6)^2 + (-2 - 5)^2 = 9 + 49 = 58$.

Midpoint (1, 3); $y = -2.5x + 5.5$; $PA = PB = \sqrt{58}$

Problem 15 Equidistant meeting point

The perpendicular bisector of AB (horizontal, from $(0, 0)$ to $(8, 0)$) is the vertical line $x = 4$. The perpendicular bisector of BC (vertical, from $(8, 0)$ to $(8, 6)$) is the horizontal line $y = 3$. They meet at $(4, 3)$.

Distance to A : $\sqrt{4^2 + 3^2} = 5$. To B : $\sqrt{4^2 + 3^2} = 5$. To C : $\sqrt{4^2 + 3^2} = 5$.

Meeting point (4, 3); 5 blocks from each home

Problem 16 Hexagonal tile

Side = 9 inches, so perimeter = 54 inches.

The apothem is the height of an equilateral triangle with side 9: $9\sqrt{3} \div 2 \approx 7.79$ inches.

Area = $\frac{1}{2} \cdot \text{apothem} \cdot \text{perimeter} = \frac{1}{2} (7.794)(54) \approx 210.4$ square inches. (Check: 6 equilateral

triangles, each $(\sqrt{3} \div 4)(81) \approx 35.07$, total ≈ 210.4 .)

Perimeter 54 in; apothem ≈ 7.79 in; area ≈ 210.4 in²

Problem 17 Square patio around a fountain

The diagonal of the square is the diameter, 13 meters, so the side is $13 \div \sqrt{2} = 6.5\sqrt{2} \approx 9.19$ meters.

Area of the patio = $2r^2 = 2(42.25) = 84.5$ square meters. Area of the circle = $\pi(42.25) \approx 132.73$.

Percent covered = $84.5 \div 132.73 \approx 0.637$, or 63.7%.

Side ≈ 9.19 m; area 84.5 m²; about 63.7% of the circle

Problem 18 Prove the angle bisector construction

Statement	Reason
1. $OX = OY$	Radii of the same arc centered at O
2. $XZ = YZ$	Arcs drawn with the same compass opening
3. $OZ = OZ$	Reflexive Property
4. $\triangle OXZ \cong \triangle OYZ$	SSS
5. $\angle XOZ \cong \angle YOZ$	CPCTC

$\angle XOZ \cong \angle YOZ$

Problem 19 Prove the perpendicular bisector construction

Statement	Reason
1. $AP = BP$ and $AQ = BQ$	Arcs drawn with the same compass opening
2. P and Q are each equidistant from A and B	Definition of equidistant
3. P and Q lie on the perpendicular bisector of AB	Converse of the Perpendicular Bisector Theorem
4. Line PQ is the perpendicular bisector of AB	Two points determine a line

PQ is the perpendicular bisector of AB

Problem 20 Prove the radius steps around a circle

Statement	Reason
1. $OA = r$ and $OB = r$	Radii of circle O
2. $AB = r$	Radius of the arc centered at A

Statement	Reason
3. $OA = OB = AB$	Substitution
4. $\triangle OAB$ is equilateral	Definition of equilateral triangle
5. $m\angle AOB = 60^\circ$	Each angle of an equilateral triangle is 60°

Since $360^\circ \div 60^\circ = 6$, six marks around the circle close it exactly.

$\angle AOB = 60^\circ$; six steps