

Solving Any Triangle with Sines, Cosines and Area

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-SRT.D.9, G-SRT.D.10, G-SRT.D.11

Each answer key entry names the law, shows the substitution and lists values before rounding. Check the side-angle pairing in the Law of Sines first when an answer differs from yours.

Standards: G-SRT.D.9, G-SRT.D.10, G-SRT.D.11 · **Level:** Grade 10 Geometry

Problem 1 Area from Two Sides

$$\text{Area} = \frac{1}{2} \cdot 16 \cdot 7 \cdot \sin 41^\circ = 56 \cdot 0.6561 = 36.74.$$

$$\approx 36.7 \text{ ft}^2$$

Problem 2 Area with an Obtuse Angle

$$\text{Area} = \frac{1}{2} \cdot 12.5 \cdot 8 \cdot \sin 115^\circ. \text{ Since } \sin 115^\circ = \sin 65^\circ \approx 0.9063, \text{ area} = 50 \cdot 0.9063 = 45.32.$$

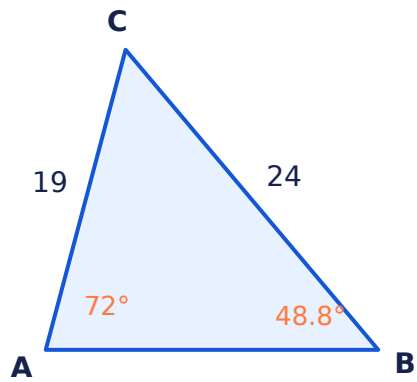
$$\approx 45.3 \text{ ft}^2$$

Problem 3 Law of Sines for a Side

$$\frac{b}{(\sin 71^\circ)} = \frac{19}{(\sin 44^\circ)}, \text{ so } b = \frac{(19 \sin 71^\circ)}{(\sin 44^\circ)} \approx \frac{17.96}{0.6947} \approx 25.86.$$

$$b \approx 25.9$$

Problem 4 Law of Sines for an Angle



Angle B is acute.

$$\frac{(\sin B)}{19} = \frac{(\sin 72^\circ)}{24}, \text{ so } \sin B = \frac{(19 \cdot 0.9511)}{24} \approx 0.7529.$$

$B = \sin^{-1}(0.7529) \approx 48.84^\circ$. The obtuse choice 131.2° is impossible because $72^\circ + 131.2^\circ$ is more than 180° .

$B \approx 48.8^\circ$

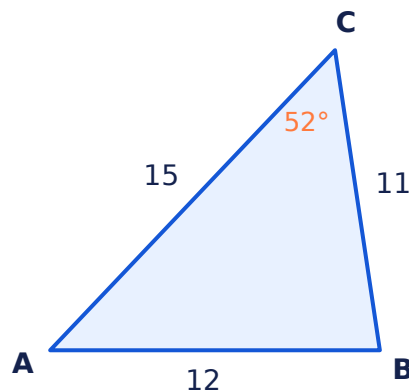
Problem 5 Find the Third Angle First

$$B = 180^\circ - 53^\circ - 71^\circ = 56^\circ.$$

$$\frac{a}{(\sin 53^\circ)} = \frac{40}{(\sin 71^\circ)}, \text{ so } a = \frac{(40 \sin 53^\circ)}{(\sin 71^\circ)} \approx 33.79.$$

$B = 56^\circ$, $a \approx 33.8$

Problem 6 Law of Cosines for a Side



Solution figure.

$$c^2 = 11^2 + 15^2 - 2(11)(15) \cos 52^\circ = 346 - 330 \cdot 0.6157 = 142.83.$$

$c = \sqrt{142.83} \approx 11.95$, which rounds to 12.0.

$c \approx 12.0$

Problem 7 Obtuse Included Angle

$\cos 124^\circ = -\cos 56^\circ \approx -0.5592$.

$c^2 = 56.25 + 100 - 150(-0.5592) = 156.25 + 83.88 = 240.13$.

$c \approx 15.5$.

$c \approx 15.5$ in

Problem 8 Largest Angle from Three Sides

The largest angle is opposite 15. $\cos \theta = \frac{(7^2 + 11^2 - 15^2)}{(2 \cdot 7 \cdot 11)} = \frac{(49 + 121 - 225)}{154} = \frac{(-55)}{154} \approx -0.3571$.

$\theta = \cos^{-1}(-0.3571) \approx 110.92^\circ$.

$\theta \approx 110.9^\circ$

Problem 9 Choose the Law

(a) Two angles and a side with its opposite angle: Law of Sines.

(b) Two sides and the included angle: Law of Cosines.

(c) Three sides: Law of Cosines.

(d) Two angles and a side with its opposite angle: Law of Sines.

(a) Sines (b) Cosines (c) Cosines (d) Sines

Problem 10 Solve from Three Sides

Largest angle C (opposite 15): $\cos C = \frac{(64 + 121 - 225)}{(2 \cdot 8 \cdot 11)} = \frac{(-40)}{176} \approx -0.2273$, so $C \approx 103.14^\circ$.

$\cos A = \frac{(121 + 225 - 64)}{(2 \cdot 11 \cdot 15)} = \frac{282}{330} \approx 0.8545$, so $A \approx 31.29^\circ$.

$B = 180^\circ - 31.29^\circ - 103.14^\circ \approx 45.57^\circ$.

$A \approx 31.3^\circ$, $B \approx 45.6^\circ$, $C \approx 103.1^\circ$

Problem 11 Solve from Two Sides and an Angle

$$a^2 = 22^2 + 17^2 - 2(22)(17) \cos 67^\circ = 773 - 748 \cdot 0.3907 = 480.73, \text{ so } a \approx 21.93.$$

$$\sin C = \frac{(17 \sin 67^\circ)}{21.93} \approx 0.7137, \text{ so } C \approx 45.54^\circ \text{ (the side 17 is not the longest, so } C \text{ is acute).}$$

$$B = 180^\circ - 67^\circ - 45.54^\circ \approx 67.46^\circ.$$

$$\mathbf{a \approx 21.9, B \approx 67.5^\circ, C \approx 45.5^\circ}$$

Problem 12 Solve from Two Angles and a Side

$$A = 180^\circ - 39^\circ - 88^\circ = 53^\circ.$$

$$b = \frac{(31 \sin 39^\circ)}{(\sin 53^\circ)} \approx 24.43. \quad c = \frac{(31 \sin 88^\circ)}{(\sin 53^\circ)} \approx 38.79.$$

$$\mathbf{A = 53^\circ, b \approx 24.4, c \approx 38.8}$$

Problem 13 Area from Three Sides

$$\text{Angle } C \text{ opposite } 16: \cos C = \frac{(81 + 169 - 256)}{(2 \cdot 9 \cdot 13)} = \frac{(-6)}{234} \approx -0.0256, \text{ so } C \approx 91.47^\circ.$$

$$\text{Area} = \frac{1}{2} \cdot 9 \cdot 13 \cdot \sin 91.47^\circ \approx 58.5 \cdot 0.9997 \approx 58.48.$$

$$\approx \mathbf{58.5 \text{ yd}^2}$$

Problem 14 Area of a Parallelogram

$$\text{Each triangle has area } \frac{1}{2} \cdot 18 \cdot 12.5 \cdot \sin 64^\circ = 101.11. \text{ Two triangles give } 202.23.$$

$$\approx \mathbf{202.2 \text{ in}^2}$$

Problem 15 Distance Across a Lake

$$AB^2 = 420^2 + 365^2 - 2(420)(365) \cos 71^\circ = 309,625 - 306,600 \cdot 0.3256 = 209806.$$

$$AB \approx 458.05.$$

$$\approx \mathbf{458 \text{ ft}}$$

Problem 16 Two Boats Leave a Dock

$$\text{The angle between the paths is } 112^\circ - 25^\circ = 87^\circ.$$

$$d^2 = 12^2 + 9^2 - 2(12)(9) \cos 87^\circ = 225 - 216 \cdot 0.0523 = 213.7.$$

$$d \approx 14.62.$$

$$\approx \mathbf{14.6 \text{ mi}}$$

Problem 17 Observers Tracking a Plane

The angle at the plane is $180^\circ - 41^\circ - 67^\circ = 72^\circ$.

(a) The distance is opposite the 67° angle: $\frac{(3.2 \sin 67^\circ)}{(\sin 72^\circ)} \approx \frac{2.95}{0.9511} \approx 3.10 \text{ mi.}$

(b) Altitude = $3.10 \cdot \sin 41^\circ \approx 2.03 \text{ mi.}$

(a) $\approx 3.10 \text{ mi}$ (b) $\approx 2.03 \text{ mi}$

Problem 18 Two Possible Triangles

$\sin B = \frac{(12 \sin 30^\circ)}{7} = \frac{6}{7} \approx 0.8571$. So $B \approx 59^\circ$ or $B \approx 121^\circ$; both fit with 30° because each sum is less than 180° .

Triangle 1: $C \approx 91^\circ$, $c = \frac{(7 \sin 91^\circ)}{(\sin 30^\circ)} \approx 14$.

Triangle 2: $C \approx 29^\circ$, $c = \frac{(7 \sin 29^\circ)}{(\sin 30^\circ)} \approx 6.79$.

Triangle 1: $C \approx 91^\circ$, $c \approx 14$. Triangle 2: $C \approx 29^\circ$, $c \approx 6.8$

Problem 19 Area of a Four-Sided Lot

Triangle 1: $\frac{1}{2} \cdot 62 \cdot 48 \cdot \sin 75^\circ = 1,488 \cdot 0.9659 = 1437.3$.

Triangle 2: $\frac{1}{2} \cdot 55 \cdot 40 \cdot \sin 110^\circ = 1,100 \cdot 0.9397 = 1033.66$.

Total = 2470.96.

$\approx 2471 \text{ ft}^2$

Problem 20 Drone Return Flight

(a) $AC^2 = 800^2 + 650^2 - 2(800)(650) \cos 128^\circ = 1,062,500 + 1,040,000 \cdot 0.6157 = 1702788$.
 $AC \approx 1304.91$.

(b) $\sin A = \frac{(650 \sin 128^\circ)}{1304.91} \approx 0.3925$, so $A \approx 23.11^\circ$ (acute, since 650 is the shorter side next to the obtuse angle).

(a) ≈ 1304.9 ft (b) $\approx 23.1^\circ$