

Parabolas from Focus and Directrix, and Tangents

Grade 10 – Geometry math · Lesson · Standards: G-GPE.A.2, G-C.A.4

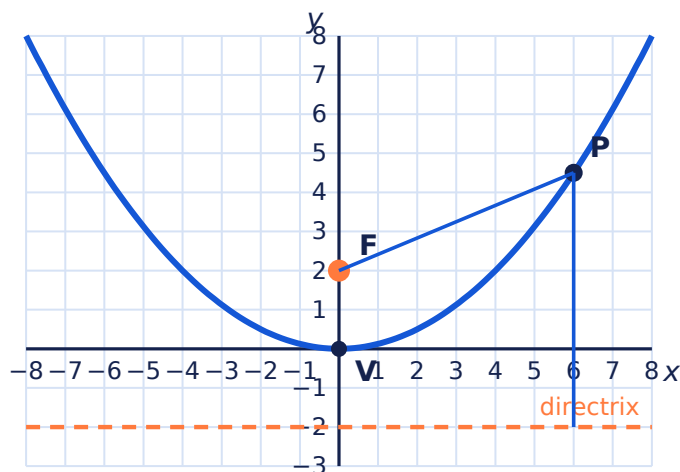
Quick answer: A parabola is the set of points equally distant from a focus and a directrix, giving equations like $x^2 = 4py$ with focus $(0, p)$. Tangent lines to a circle are perpendicular to the radius at the point of contact, and the two tangent segments from an outside point are congruent.

Standards: G-GPE.A.2, G-C.A.4 · **Level:** Grade 10 Geometry

What is a parabola?

Parabola

Take one point (the focus) and one line (the directrix) that does not contain it. The parabola collects every point whose two distances, one to the focus and one to the line, come out equal. Its vertex sits midway between the focus and the line.



P is 6.5 from F and 6.5 from the directrix: $x^2 = 8y$.

How do you get the equation from the focus and directrix?

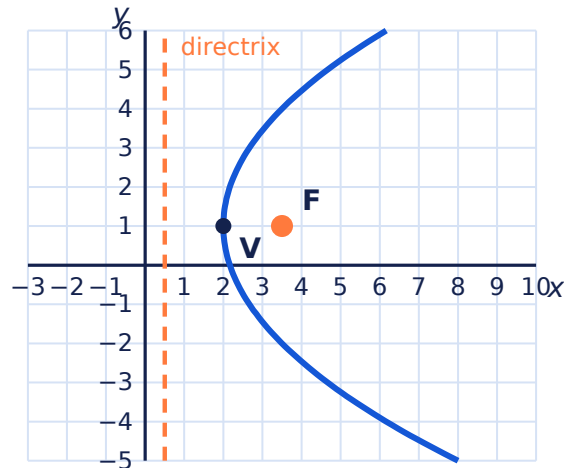
Place the focus at $(0, p)$ and the directrix along $y = -p$. A point (x, y) belongs to the parabola exactly when these two distances match:

$$\sqrt{x^2 + (y - p)^2} = y + p$$

After squaring, the y^2 and p^2 terms cancel and $x^2 = 4py$ remains. A larger p gives a wider, flatter curve.

Standard forms with vertex (h, k)

- Vertical axis: $(x - h)^2 = 4p(y - k)$, with focus $(h, k + p)$ and directrix $y = k - p$.
- Horizontal axis: $(y - k)^2 = 4p(x - h)$, with focus $(h + p, k)$ and directrix $x = h - p$.
- When p is negative the curve opens the opposite way.

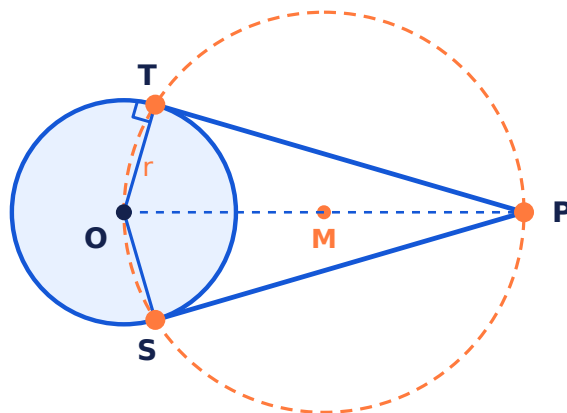


$$(y - 1)^2 = 6(x - 2): \text{vertex } (2, 1), p = 1.5.$$

How do tangent lines to a circle behave?

Tangent facts

- The radius drawn to the touching point makes a right angle with the tangent.
- From any outside point, the two tangent segments have equal length.
- With center O and touching point T , $PT^2 = OP^2 - r^2$.



Construction: the circle on diameter OP crosses the circle at the tangent points.

Method: construct a tangent from an outside point

1. Join the center O to the outside point P .
2. Bisect OP with a perpendicular bisector to mark its midpoint M .
3. With the compass at M , draw the circle that passes through both O and P .
4. The new circle crosses the original at T and S ; draw PT and PS .

Reason: OP is a diameter of the helper circle, so $\angle OTP$ is an inscribed angle on a semicircle and equals 90° . That makes $PT \perp OT$.

Example

A parabola has focus $(0, 3)$ and directrix $y = -3$. Here $p = 3$, so $x^2 = 12y$. The point $(6, 3)$ satisfies $36 = 12 \cdot 3$, and its distance to the focus (6) equals its distance to the directrix (6) .

Common mistake

Using $4p$ as the distance to the focus. The distance from vertex to focus is p , so in $x^2 = 12y$ the focus is 3 units from the vertex.

Mathbot's tip Match the number in front of the unsquared variable to $4p$, then divide by 4. The sign tells you the direction.

Key points to remember

- Parabola: distance to the focus equals distance to the directrix.
- Opening up or down: $(x - h)^2 = 4p(y - k)$, focus $(h, k + p)$, directrix $y = k - p$.
- Opening right or left: $(y - k)^2 = 4p(x - h)$, focus $(h + p, k)$, directrix $x = h - p$.
- Two tangent segments from an outside point are congruent, and $PT^2 = OP^2 - r^2$.

Parabolas from Focus and Directrix, and Tangents: frequently asked questions

What is the focus of a parabola used for?

Rays parallel to the axis reflect off a parabolic dish and meet at the focus. That is why satellite dishes and headlights are shaped like parabolas.

How do I find p from an equation?

Write the equation as $(x - h)^2 = c(y - k)$ or $(y - k)^2 = c(x - h)$. Then $4p$ equals c , so $p = c \div 4$. A negative c means the parabola opens down or left.

Where is the vertex relative to the focus and directrix?

The vertex is exactly halfway between them, on the axis of the parabola. The distance from the vertex to each is p .

Why are two tangent segments from one point congruent?

The two right triangles formed with the center share the hypotenuse OP and have equal radii as legs, so by the Pythagorean Theorem or HL the tangent segments are equal.

How do I construct a tangent from a point outside a circle?

Find the midpoint of the segment joining the center and the point, draw a circle centered there through both, and connect the outside point to the two places where the circles intersect.