

Parabolas from Focus and Directrix, and Tangents

Grade 10 – Geometry math · Test Answer Key · Standards: G-GPE.A.2, G-C.A.4

Each solution shows the vertex, p , the equation and the focus or length requested. The scoring notes show how partial credit is earned for each step.

Standards: G-GPE.A.2, G-C.A.4 · **Level:** Grade 10 Geometry

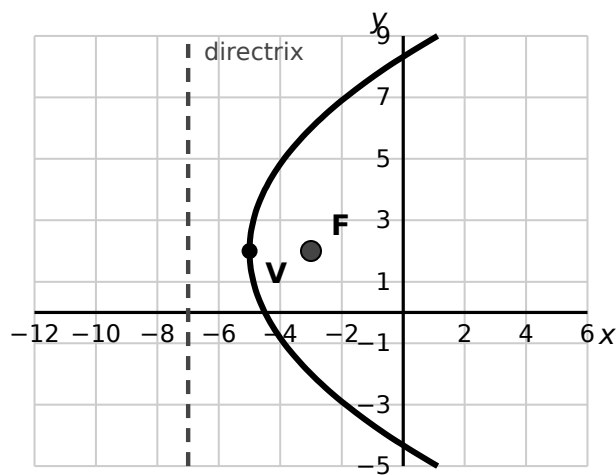
Question 1 Equation from the Focus (2 points)

The vertex is $(0, 0)$ and $p = 10$, so $x^2 = 40y$.

$$x^2 = 40y$$

Scoring: 1 point for p , 1 point for the equation.

Question 2 Parabola Opening Left or Right (3 points)



Vertex $(-5, 2)$.

The vertex is halfway: $(-5, 2)$. $p = -3 - (-5) = 2$, so the parabola opens right.

$$(y - 2)^2 = 4(2)(x + 5).$$

$$(y - 2)^2 = 8(x + 5)$$

Scoring: 1 point for the vertex, 1 point for p , 1 point for the equation.

Question 3 Complete the Square (3 points)

$x^2 + 10x = -12y - 49$. Add 25: $(x + 5)^2 = -12y - 24 = -12(y + 2)$.

Vertex $(-5, -2)$, $p = -3$. Focus $(-5, -5)$. Directrix $y = 1$.

$(x + 5)^2 = -12(y + 2)$; focus $(-5, -5)$; directrix $y = 1$

Scoring: 1 point for completing the square, 1 point for the focus, 1 point for the directrix.

Question 4 Is the Point on the Parabola? (2 points)

Distance from P to the focus $(0, 5)$: 10. Distance from P to the directrix $y = -5$: $5 + 5 = 10$.

Yes, P is on the parabola.

Scoring: 1 point for each distance.

Question 5 Tangent Segment Length (2 points)

$OT \perp PT$, so $PT = \sqrt{(37^2 - 12^2)} = \sqrt{1225} = 35$.

35

Scoring: 1 point for the right triangle, 1 point for the answer.

Question 6 Tangent Segments with Algebra (3 points)

$4x - 9 = 2x + 11$, so $2x = 20$ and $x = 10$. $PA = 31$.

$OP = \sqrt{(31^2 + 12^2)} = \sqrt{1105} \approx 33.24$.

$x = 10$; $PA = 31$; $OP \approx 33.2$

Scoring: 1 point for x , 1 point for PA , 1 point for OP .

Question 7 Telescope Dish (3 points)

The edge point is $(18, 6)$. $x^2 = 4py$ gives $324 = 4p(6) = 24p$, so $p = 13.5$.

13.5 inches above the vertex

Scoring: 1 point for the edge point, 1 point for the equation, 1 point for p .

Question 8 Why the Construction Works (2 points)

$\angle OTP$ is inscribed in a semicircle of the circle with diameter OP , so it is a right angle. A line perpendicular to a radius at its endpoint on the circle is tangent.

$\angle OTP = 90^\circ$, so $PT \perp OT$ and PT is tangent.

Scoring: 1 point for the semicircle angle, 1 point for the perpendicular-radius conclusion.