

Parabolas from Focus and Directrix, and Tangents

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-GPE.A.2, G-C.A.4

Each solution identifies the vertex and p , writes the equation, and lists the focus and directrix. For tangents, check for the right angle with the Pythagorean Theorem before accepting a length.

Standards: G-GPE.A.2, G-C.A.4 · **Level:** Grade 10 Geometry

Problem 1 Focus and Directrix Distances

(a) Distance to focus: 6. Distance to the directrix: $3 - (-3) = 6$. Equal, so on the parabola.

(b) Distance to focus: $\sqrt{(4^2 + 1^2)} = \sqrt{17} \approx 4.12$. Distance to directrix: $2 + 3 = 5$. Not equal.

(a) on the parabola (b) not on the parabola

Problem 2 Equation with Focus Above

The vertex is halfway between the focus and directrix, at $(0, 0)$, and $p = 6$, so $x^2 = 4(6)y$.

$$x^2 = 24y$$

Problem 3 Equation Opening Downward

The vertex is $(0, 0)$ and the focus is below it, so $p = -7$ and $x^2 = 4(-7)y$.

$$x^2 = -28y$$

Problem 4 Read the Focus

$4p = 36$, so $p = 9$. The focus is $(0, 9)$ and the directrix is $y = -9$.

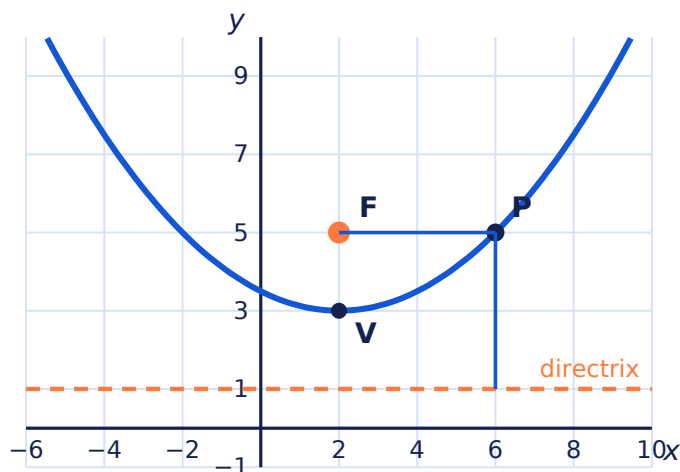
Focus $(0, 9)$; directrix $y = -9$

Problem 5 Parabola Opening Right

The y is squared and the coefficient is positive, so it opens right. $4p = 24$, so $p = 6$. The focus is $(6, 0)$ and the directrix is $x = -6$.

Opens right; focus (6, 0); directrix $x = -6$

Problem 6 Vertex Form from Focus and Directrix



Vertex (2, 3); the point (6, 5) is 4 from the focus and 4 from the directrix.

The vertex is halfway between: (2, 3). The focus is 2 above the vertex, so $p = 2$.

$$(x - 2)^2 = 4(2)(y - 3).$$

$$(x - 2)^2 = 8(y - 3)$$

Problem 7 Read a Shifted Parabola

The vertex is $(-1, 4)$. $4p = -12$, so $p = -3$ and the parabola opens down.

Focus: $(-1, 4 - 3) = (-1, 1)$. Directrix: $y = 4 + 3 = 7$.

Vertex $(-1, 4)$; focus $(-1, 1)$; directrix $y = 7$

Problem 8 Equation Opening Right

The vertex is halfway between the focus and the directrix on the line $y = -2$: $x = \frac{(3 + (-1))}{2} = 1$, so the vertex is $(1, -2)$. $p = 3 - 1 = 2$.

$$(y + 2)^2 = 4(2)(x - 1).$$

$$(y + 2)^2 = 8(x - 1)$$

Problem 9 Derive the Equation

For a point (x, y) : $\sqrt{(x^2 + (y - 3)^2)} = y + 3$.

Square both sides: $x^2 + y^2 - 6y + 9 = y^2 + 6y + 9$.

Cancel y^2 and 9: $x^2 = 12y$.

$x^2 = 12y$

Problem 10 A Point on the Parabola

$36 = 8y$, so $y = 4.5$.

The focus is $(0, 2)$: distance $= \sqrt{(6^2 + 2.5^2)} = \sqrt{42.25} = 6.5$. The directrix is $y = -2$: distance $= 4.5 + 2 = 6.5$.

$y = 4.5$; both distances are 6.5

Problem 11 Width at the Focus

$4p = 20$, so $p = 5$ and the focus is $(0, 5)$.

At $y = 5$: $x^2 = 100$, so $x = \pm 10$. The width is 20.

Focus $(0, 5)$; width 20 (equal to $4p$)

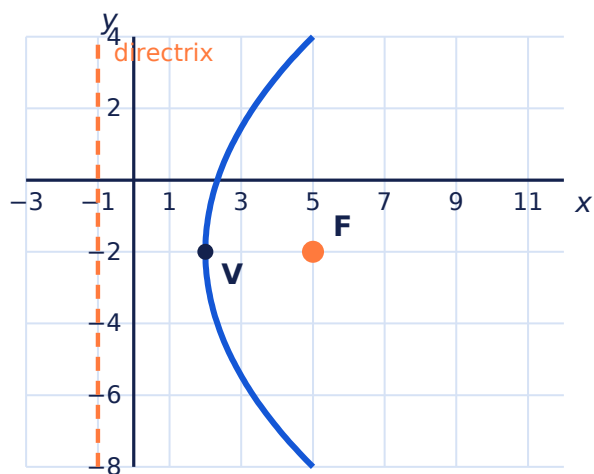
Problem 12 Complete the Square, Opening Up

Isolate the x-terms: $x^2 - 6x = 8y - 17$. Add 9: $(x - 3)^2 = 8y - 8 = 8(y - 1)$.

Vertex $(3, 1)$, $4p = 8$ so $p = 2$. Focus $(3, 3)$. Directrix $y = -1$.

Vertex $(3, 1)$; focus $(3, 3)$; directrix $y = -1$

Problem 13 Complete the Square, Opening Right



Vertex $(2, -2)$, focus $(5, -2)$.

$y^2 + 4y = 12x - 28$. Add 4: $(y + 2)^2 = 12x - 24 = 12(x - 2)$.

Vertex $(2, -2)$, $p = 3$. Focus $(5, -2)$. Directrix $x = -1$.

Vertex $(2, -2)$; focus $(5, -2)$; directrix $x = -1$

Problem 14 Congruent Tangent Segments

Tangent segments from one point are congruent: $6x - 5 = 4x + 13$, so $2x = 18$ and $x = 9$.

$PA = 6(9) - 5 = 49$. Check: $PB = 4(9) + 13 = 49$.

$x = 9$; $PA = PB = 49$

Problem 15 Rope to the Pond

The rope is tangent, so $OT \perp PT$ and triangle OTP is right with hypotenuse 25.

$PT = \sqrt{(25^2 - 7^2)} = \sqrt{576} = 24$.

24 ft

Problem 16 Is the Line Tangent?

(a) $9^2 + 40^2 = 81 + 1,600 = 1,681 = 41^2$. The converse of the Pythagorean Theorem gives a right angle at T : tangent.

(b) $8^2 + 14^2 = 64 + 196 = 260$, but $16^2 = 256$. Not equal, so no right angle: not tangent.

(a) tangent (b) not tangent

Problem 17 Satellite Dish Focus

The edge of the dish is at (12, 4.5). Use $x^2 = 4py$: $144 = 4p(4.5) = 18p$, so $p = 8$.

The receiver sits 8 feet above the vertex.

Problem 18 Headlight Reflector

(a) $4p = 12$, so $p = 3$. The bulb is 3 in above the vertex.

(b) At the edge $x = 8$: $y = \frac{64}{12} = \frac{16}{3} \approx 5.33$.

(a) 3 in (b) $16/3 \approx 5.33$ in deep

Problem 19 Construct and Justify

(a) $\triangle OTP$ is inscribed in a semicircle (its sides pass through the ends of the diameter OP), so $\angle OTP = 90^\circ$. Then PT is perpendicular to the radius OT at T , which makes it tangent.

(b) $PT = \sqrt{(11^2 - 6^2)} = \sqrt{85} \approx 9.22$.

(b) $PT \approx 9.22$ cm

Problem 20 Angle Between Two Tangents

The line from P to the center bisects the 70° angle, so the angle at P in the right triangle is 35° .

$OP = \frac{8}{(\sin 35^\circ)} \approx 13.95$. $PT = \frac{8}{(\tan 35^\circ)} \approx 11.43$.

$OP \approx 13.95$ in; $PT \approx 11.43$ in