

Parabolas from Focus and Directrix, and Tangents

Grade 10 – Geometry math · Worksheet · Standards: G-GPE.A.2, G-C.A.4

Name: _____ Date: _____ Class: _____

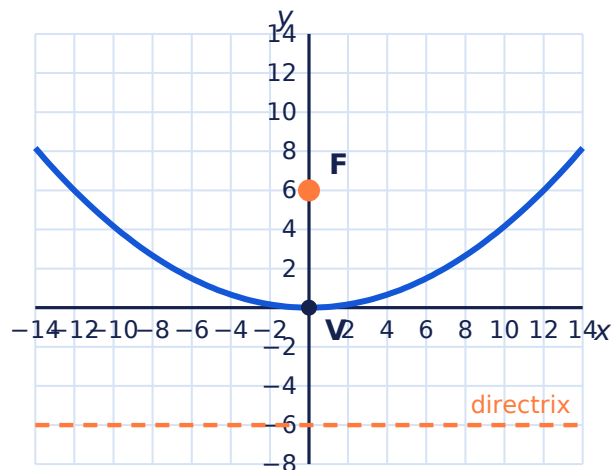
These problems derive and use the equation of a parabola from its focus and directrix and apply the properties of tangent lines to a circle (G-GPE.A.2 and G-C.A.4). Early problems read and write standard form; later problems use dishes, ropes and constructions. Satellite dishes, headlights and ponds supply the contexts.

Standards: G-GPE.A.2, G-C.A.4 · **Level:** Grade 10 Geometry

Problem 1 Focus and Directrix Distances (easy)

A parabola has focus $(0, 3)$ and directrix $y = -3$. Decide whether each point is on the parabola by comparing distances: (a) $(6, 3)$ (b) $(4, 2)$.

Problem 2 Equation with Focus Above (easy)



Focus $(0, 6)$, directrix $y = -6$.

A designer sketches a parabola whose focus is $(0, 6)$ and whose directrix is the line $y = -6$. State its equation.

Problem 3 Equation Opening Downward (medium)

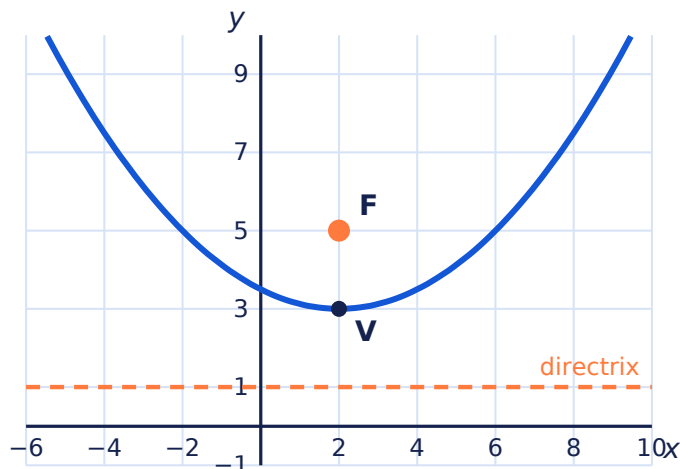
A parabolic arch opens downward with focus $(0, -7)$ and directrix $y = 7$. Write its equation.

Problem 4 Read the Focus (easy)

A mirror profile is the parabola $x^2 = 36y$. Locate its focus and its directrix.

Problem 5 Parabola Opening Right (medium)

A sideways parabola is given by $y^2 = 24x$. State the direction it opens, its focus and its directrix.

Problem 6 Vertex Form from Focus and Directrix (medium)

Focus (2, 5), directrix $y = 1$.

Write the equation of the parabola with focus (2, 5) and directrix $y = 1$.

Problem 7 Read a Shifted Parabola (medium)

Find the vertex, focus and directrix of $(x + 1)^2 = -12(y - 4)$.

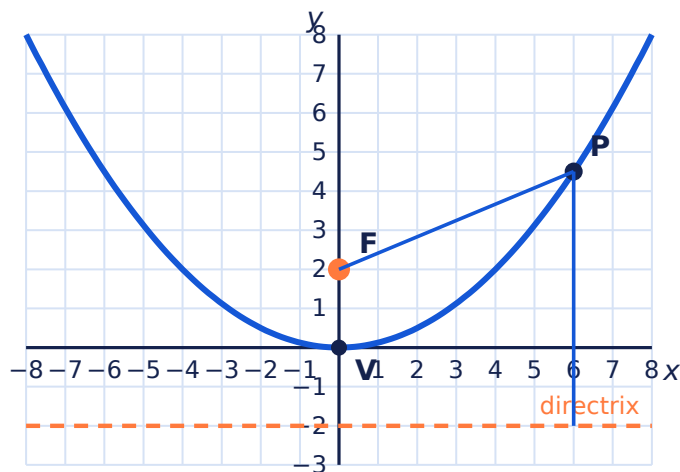
Problem 8 Equation Opening Right (hard)

Write the equation of the parabola with focus (3, -2) and directrix $x = -1$.

Problem 9 Derive the Equation (hard)

A parabola has focus (0, 3) and directrix $y = -3$. Start from "distance to the focus equals distance to the directrix" and derive its equation.

Problem 10 A Point on the Parabola (medium)



Parabola $x^2 = 8y$.

The point $(6, y)$ lies on the parabola $x^2 = 8y$. Find y , then verify that the point is the same distance from the focus and from the directrix.

Problem 11 Width at the Focus (hard)

A parabola has equation $x^2 = 20y$. Find its focus and the width of the parabola at the height of the focus.

Problem 12 Complete the Square, Opening Up (hard)

Rewrite $x^2 - 6x - 8y + 17 = 0$ in vertex form. Give the vertex, focus and directrix.

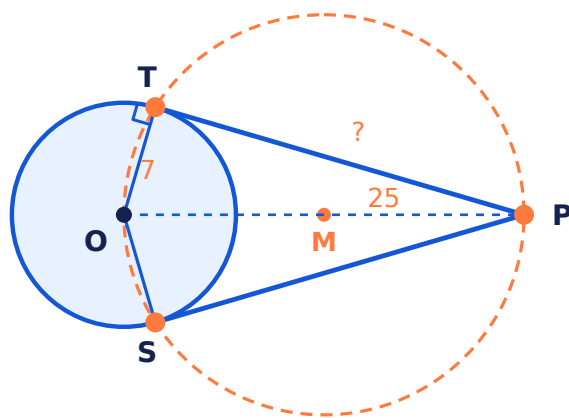
Problem 13 Complete the Square, Opening Right (hard)

Rewrite $y^2 + 4y - 12x + 28 = 0$ in vertex form. Give the vertex, focus and directrix.

Problem 14 Congruent Tangent Segments (hard)

Tangent segments PA and PB are drawn from point P to a circle. $PA = 6x - 5$ and $PB = 4x + 13$. Find x and the length of each segment.

Problem 15 Rope to the Pond (hard)



Pond radius 7 ft, $OP = 25$ ft.

A circular pond has a radius of 7 feet. A post is at point P , 25 feet from the center. A rope from the post touches the pond at exactly one point T . How long is the rope?

Problem 16 Is the Line Tangent? (hard)

Segment PT touches a circle with center O at point T . Decide whether PT is tangent in each case by testing for a right angle at T . (a) $OT = 9$, $PT = 40$, $OP = 41$. (b) $OT = 8$, $PT = 14$, $OP = 16$.

Problem 17 Satellite Dish Focus (hard)

A satellite dish has a parabolic cross-section with its vertex at the origin, opening up. It is 24 feet wide and 4.5 feet deep. Where is the focus, measured above the vertex?

Problem 18 Headlight Reflector (hard)

A reflector's cross-section is the parabola $x^2 = 12y$ (units in inches) with the bulb at the focus. (a) How far above the vertex is the bulb? (b) The reflector opening is 16 inches wide. How deep is the reflector?

Problem 19 Construct and Justify (hard)

Circle O has radius 6 centimeters and point P is 11 centimeters from O . A student draws the circle with diameter OP , which meets the first circle at T . (a) Explain why PT is tangent. (b) Find PT to the nearest hundredth.

Problem 20 Angle Between Two Tangents (hard)

Two tangents from point P to a circle of radius 8 inches form a 70° angle. Find the distance from

P to the center and the length of each tangent segment, to the nearest hundredth.