

Parallel Lines Cut by a Transversal

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-CO.C.9

Each answer names the angle relationship, writes the equation and checks the result. In the proofs, every reason cites a theorem, a definition or a property. Compare the chain of statements with your own.

Standards: G-CO.C.9 · **Level:** Grade 10 Geometry

Problem 1 Name the angle pairs

- Between the lines, opposite sides of the transversal: alternate interior.
- Same position at the two intersections: corresponding.
- Between the lines, same side of the transversal: same-side interior.
- Outside the lines, opposite sides: alternate exterior.

(a) alternate interior, (b) corresponding, (c) same-side interior, (d) alternate exterior

Problem 2 Alternate interior and same-side

$\angle 3$ and $\angle 6$ are alternate interior angles, so $m\angle 6 = 74^\circ$. $\angle 3$ and $\angle 5$ are same-side interior angles, so $m\angle 5 = 180^\circ - 74^\circ = 106^\circ$.

$m\angle 6 = 74^\circ$, $m\angle 5 = 106^\circ$

Problem 3 Same-side interior angles

$\angle 4$ and $\angle 6$ are same-side interior angles, so they add to 180° . Then $m\angle 6 = 180^\circ - 127^\circ = 53^\circ$.

53°

Problem 4 Alternate exterior angles

$\angle 1$ and $\angle 8$ are alternate exterior angles, so $m\angle 8 = 133^\circ$. $\angle 2$ forms a linear pair with $\angle 1$, so $m\angle 2 = 47^\circ$, and $\angle 7$ is the alternate exterior partner of $\angle 2$, so $m\angle 7 = 47^\circ$.

$m\angle 8 = 133^\circ$, $m\angle 7 = 47^\circ$

Problem 5 Corresponding angles with algebra

Corresponding angles are congruent: $7x - 8 = 4x + 37$, so $3x = 45$ and $x = 15$.

Each angle is $7(15) - 8 = 97^\circ$, and $4(15) + 37 = 97^\circ$.

$x = 15$; both angles measure 97°

Problem 6 Same-side interior with algebra

Same-side interior angles are supplementary: $(4x + 12) + (6x - 2) = 180$, so $10x + 10 = 180$ and $x = 17$.

The angles are $4(17) + 12 = 80^\circ$ and $6(17) - 2 = 100^\circ$. Check: $80 + 100 = 180$.

$x = 17$; 80° and 100°

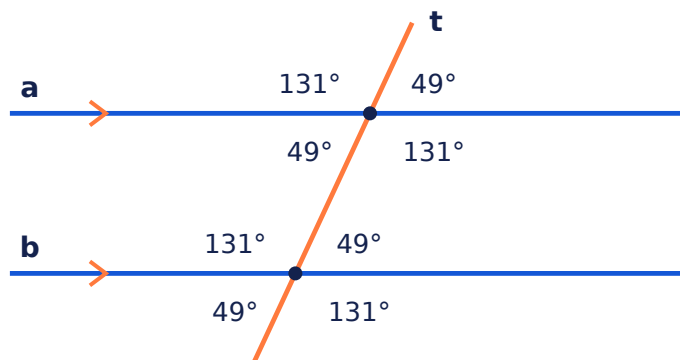
Problem 7 Alternate interior with algebra

Alternate interior angles are congruent: $2x + 31 = 5x - 17$, so $48 = 3x$ and $x = 16$.

Each angle is $2(16) + 31 = 63^\circ$, and $5(16) - 17 = 63^\circ$.

$x = 16$; 63°

Problem 8 All eight angles



All eight angles for Problem 8.

$\angle 2$ and $\angle 7$ are congruent to $\angle 6$ (corresponding or vertical), and $\angle 3$ is its alternate interior partner, so these measure 49° . The remaining angles each form a linear pair with a 49° angle: $180^\circ - 49^\circ = 131^\circ$.

$\angle 2, \angle 3, \angle 7 = 49^\circ$; $\angle 1, \angle 4, \angle 5, \angle 8 = 131^\circ$

Problem 9 Are the lines parallel

- Equal corresponding angles: the lines are parallel.
- $98^\circ + 92^\circ = 190^\circ$, not 180° , so the lines are not parallel.

c. Equal alternate exterior angles: the lines are parallel.

(a) parallel, (b) not parallel, (c) parallel

Problem 10 Value of x for parallel lines

For parallel lines the corresponding angles must be equal: $5x + 4 = 7x - 26$, so $30 = 2x$ and $x = 15$.

Each angle is $5(15) + 4 = 79^\circ$, and $7(15) - 26 = 79^\circ$.

$x = 15$; 79°

Problem 11 Same-side interior converse

The angles must be supplementary: $6x + 7 + 4x + 13 = 180$, so $10x + 20 = 180$ and $x = 16$.

The angles measure $6(16) + 7 = 103^\circ$ and $4(16) + 13 = 77^\circ$, and $103 + 77 = 180$.

$x = 16$

Problem 12 Perpendicular bisector equation

By the Perpendicular Bisector Theorem, $PA = PB$: $5x - 14 = 3x + 10$, so $2x = 24$ and $x = 12$.

$PA = 5(12) - 14 = 46$, and $PB = 3(12) + 10 = 46$.

$x = 12$; $PA = 46$

Problem 13 Alternate interior proof

Statement	Reason
1. $a \parallel b$	Given
2. $\angle 2 \cong \angle 6$	Corresponding angles are congruent (parallel lines)
3. $\angle 2 \cong \angle 3$	Vertical angles are congruent
4. $\angle 3 \cong \angle 6$	Transitive Property of congruence

$\angle 3 \cong \angle 6$

Problem 14 Prove lines parallel

Statement	Reason
1. $\angle 4$ and $\angle 6$ are supplementary	Given

Statement	Reason
2. $\angle 2$ and $\angle 4$ form a linear pair	Definition of linear pair
3. $\angle 2$ and $\angle 4$ are supplementary	Linear pairs are supplementary
4. $\angle 2 \cong \angle 6$	Angles supplementary to the same angle ($\angle 4$) are congruent
5. $a \parallel b$	Converse of the Corresponding Angles Theorem

$a \parallel b$

Problem 15 Crossing streets

- This angle is corresponding to the 71° angle, so it measures 71° .
- It forms a linear pair with the angle in part (a): $180^\circ - 71^\circ = 109^\circ$.
- It is vertical to the 71° angle, so it measures 71° .

(a) 71° , (b) 109° , (c) 71°

Problem 16 Equidistant lamp post

Equidistance gives $3x + 8 = 5x - 6$, so $14 = 2x$ and $x = 7$. Then $LB = LF = 29$ feet.

The midpoint M is 20 feet from each end, and $LM \perp BF$, so the Pythagorean Theorem gives $LM = \sqrt{(29^2 - 20^2)} = \sqrt{441} = 21$.

$x = 7$; $LB = 29$ ft; the lamp post is 21 ft from the midpoint

Problem 17 Same-side exterior angles

$\angle 1$ and $\angle 3$ form a linear pair, so they are supplementary. Because the lines are parallel, $\angle 3 \cong \angle 7$ (corresponding angles). Substituting, $\angle 1$ and $\angle 7$ are supplementary.

$(8x + 4) + (4x + 8) = 180$ gives $12x + 12 = 180$, so $x = 14$.

$\angle 1 = 8(14) + 4 = 116^\circ$ and $\angle 7 = 4(14) + 8 = 64^\circ$.

$x = 14$; 116° and 64°

Problem 18 Bisecting an angle between parallels

Alternate interior angles are equal: $5x + 3 = 7x - 25$, so $28 = 2x$ and $x = 14$. Then $m\angle 3 = 5(14) + 3 = 73^\circ$.

$\angle 4$ forms a linear pair with $\angle 3$, so $m\angle 4 = 180^\circ - 73^\circ = 107^\circ$. A bisector gives halves of $107^\circ \div 2 = 53.5^\circ$.

$$x = 14; m\angle 4 = 107^\circ; \text{ each half is } 53.5^\circ$$

Problem 19 Supplementary angles proof

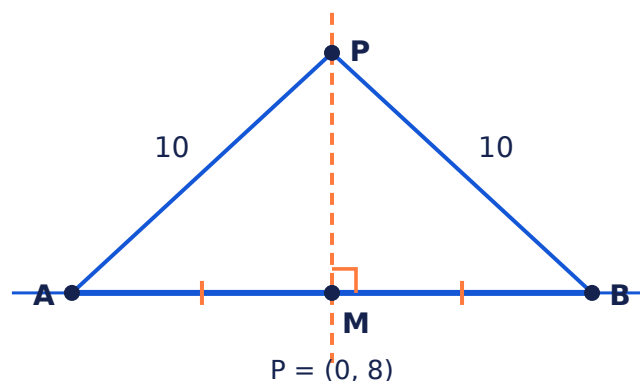
Statement	Reason
1. $a \parallel b$	Given
2. $\angle 2 \cong \angle 6$, so $m\angle 2 = m\angle 6$	Corresponding angles are congruent
3. $\angle 1$ and $\angle 2$ form a linear pair	Definition of linear pair
4. $m\angle 1 + m\angle 2 = 180^\circ$	Linear pairs are supplementary
5. $m\angle 1 + m\angle 6 = 180^\circ$	Substitution
6. $\angle 1$ and $\angle 6$ are supplementary	Definition of supplementary

$\angle 1$ and $\angle 6$ are supplementary

Problem 20 Prove the bisector theorem

Statement	Reason
1. ℓ is the perpendicular bisector of AB at M	Given
2. $AM = MB$	Definition of midpoint
3. $\angle PMA$ and $\angle PMB$ are right angles, so $\angle PMA \cong \angle PMB$	Definition of perpendicular; all right angles are congruent
4. $PM = PM$	Reflexive Property
5. $\triangle PMA \cong \triangle PMB$	SAS (two sides and the included angle)
6. $PA = PB$	Corresponding parts of congruent triangles

Check: $PA = \sqrt{((0 + 6)^2 + (8 - 0)^2)} = \sqrt{100} = 10$ and $PB = \sqrt{((0 - 6)^2 + 8^2)} = 10$.



Check with coordinates: $A(-6, 0)$, $B(6, 0)$, $P(0, 8)$.

$$\mathbf{PA = PB = 10}$$