

Quadrilateral Properties: Parallelograms to Kites

Grade 10 – Geometry math · Lesson · Standards: G-CO.C.11

Quick answer: A parallelogram has two pairs of parallel sides, so opposite sides and opposite angles are congruent, consecutive angles are supplementary, and the diagonals bisect each other. A quadrilateral is a parallelogram if its opposite sides or angles are congruent, its diagonals bisect each other, or one pair of sides is parallel and congruent. Rectangles, rhombuses, trapezoids and kites add properties.

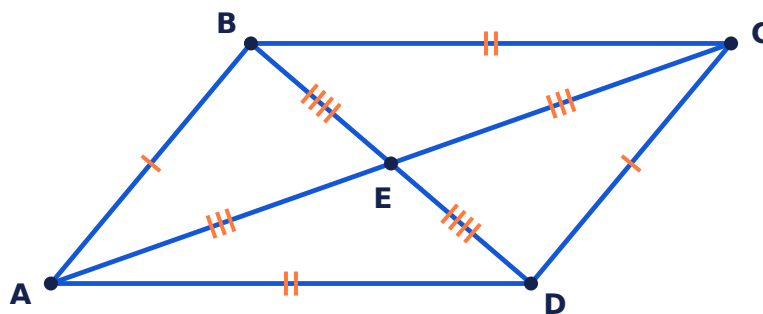
Standards: G-CO.C.11 · **Level:** Grade 10 Geometry

What is true in every parallelogram?

A **parallelogram** is a quadrilateral with two pairs of parallel sides. That definition guarantees four more facts.

Parallelogram Theorems

- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.



Parallelogram: opposite sides congruent, opposite angles congruent, diagonals bisect each other.

How do you prove that a quadrilateral is a parallelogram?

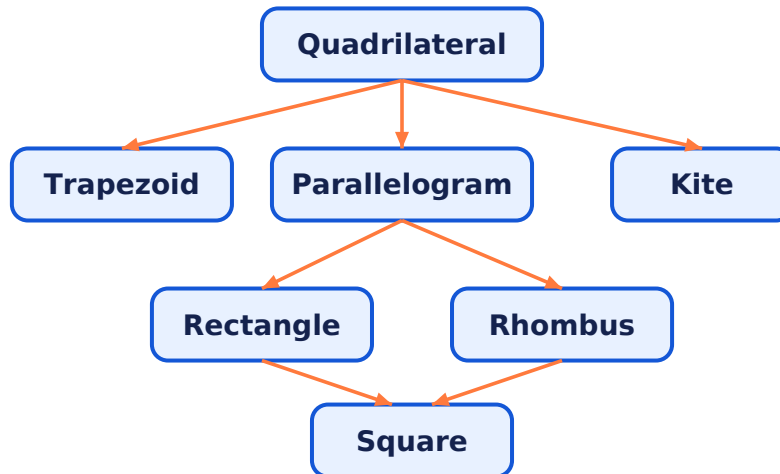
The converses work, and one more test is available. A quadrilateral is a parallelogram if any one of these holds:

- both pairs of opposite sides are congruent;

- both pairs of opposite angles are congruent;
- the diagonals bisect each other;
- one pair of opposite sides is both parallel and congruent.

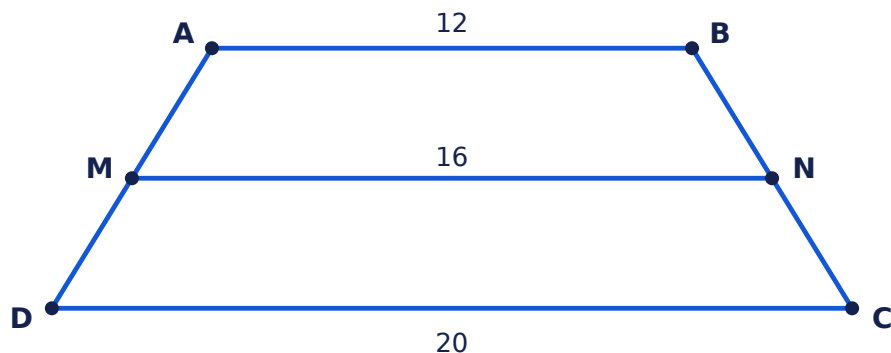
Beware: one pair of congruent sides and one pair of parallel sides is *not* enough, because an isosceles trapezoid has that pattern.

How are special quadrilaterals related?

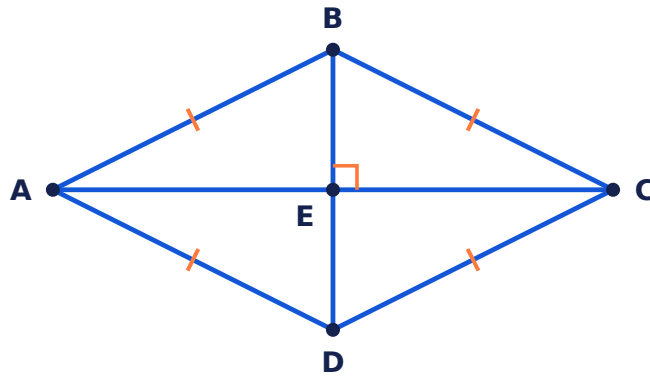


Each shape inherits all properties of the shapes above it.

Shape	Extra properties
Rectangle	Four right angles; diagonals are congruent
Rhombus	Four congruent sides; diagonals are perpendicular and bisect the angles
Square	Both a rectangle and a rhombus
Isosceles trapezoid	One pair of parallel sides; legs congruent; diagonals congruent
Kite	Two pairs of adjacent congruent sides; diagonals are perpendicular



$$\text{Trapezoid midsegment} = \frac{1}{2}(\text{base}_1 + \text{base}_2) = \frac{1}{2}(12 + 20) = 16.$$



Rhombus: diagonals are perpendicular and bisect the angles.

Method for a missing measure

1. Name the shape and list its properties.
2. Pick the property that connects the given and unknown parts.
3. Write an equation and solve it.
4. Check against a second property, such as angles adding to 180° .

Worked example

In parallelogram $ABCD$, $m\angle A = (3x + 10)^\circ$ and $m\angle B = (5x - 30)^\circ$. Consecutive angles are supplementary, so $8x - 20 = 180$ and $x = 25$. Then $\angle A = \mathbf{85^\circ}$ and $\angle B = \mathbf{95^\circ}$.

Common mistake

Do not assume that the diagonals of every parallelogram are congruent or perpendicular. Those facts belong only to rectangles and rhombuses.

Mathbot's tip

Think of the family tree: whatever is true for a parallelogram is true for rectangles, rhombuses and squares below it.

Key points to remember

- Parallelograms: opposite sides and angles congruent, diagonals bisect each other.
- Four tests prove a quadrilateral is a parallelogram, including diagonals that bisect each other.
- Rectangle diagonals are congruent; rhombus diagonals are perpendicular.
- A trapezoid midsegment equals the average of the two bases.
- A kite has two pairs of adjacent congruent sides and perpendicular diagonals.

Quadrilateral Properties: Parallelograms to Kites: frequently asked questions

How do you know a quadrilateral is a parallelogram?

Show any one test: both pairs of opposite sides congruent, both pairs of opposite angles congruent, diagonals that bisect each other, or one pair of sides that is parallel and congruent.

Is a square a rectangle?

Yes. A square has four right angles, so it is a rectangle. It also has four congruent sides, so it is a rhombus as well.

Why are consecutive angles of a parallelogram supplementary?

Consecutive angles are same-side interior angles formed by parallel sides and a transversal side. Same-side interior angles of parallel lines add to 180° .

What is the midsegment of a trapezoid?

It joins the midpoints of the two legs. It is parallel to the bases, and its length equals half the sum of the base lengths.

What distinguishes a kite from a rhombus?

A kite has two pairs of adjacent congruent sides, while a rhombus has four congruent sides. Every rhombus is a kite, but most kites are not rhombuses.