

Quadrilateral Properties: Parallelograms to Kites

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-CO.C.11

Each solution names the property used, writes the equation and checks against a second property. Proofs list a reason beside every statement, so compare your reasons as well as the final conclusion.

Standards: G-CO.C.11 · **Level:** Grade 10 Geometry

Problem 1 Opposite sides of a parallelogram

Opposite sides are congruent: $CD = AB = 14.5$ and $AD = BC = 9$.

Perimeter = $2(14.5 + 9) = 47$.

CD = 14.5, AD = 9, perimeter = 47

Problem 2 Angles of a parallelogram

Opposite angles are congruent, so $m\angle C = 68^\circ$. Consecutive angles are supplementary, so $m\angle B = 180^\circ - 68^\circ = 112^\circ$, and $m\angle D = 112^\circ$.

$\angle B = 112^\circ$, $\angle C = 68^\circ$, $\angle D = 112^\circ$

Problem 3 Diagonals that bisect

The diagonals bisect each other, so $CE = AE = 6.5$ and $DE = BE = 9.25$.

$AC = 2(6.5) = 13$ and $BD = 2(9.25) = 18.5$.

CE = 6.5, DE = 9.25, AC = 13, BD = 18.5

Problem 4 Opposite sides with algebra

Opposite sides are congruent: $4x - 7 = 2x + 9$, so $2x = 16$ and $x = 8$.

$AB = 4(8) - 7 = 25$, and $CD = 2(8) + 9 = 25$.

$x = 8$; AB = 25

Problem 5 Consecutive angles with algebra

Consecutive angles are supplementary: $(4x + 6) + (6x - 16) = 180$, so $10x - 10 = 180$ and $x = 19$.

$\angle A = 4(19) + 6 = 82^\circ$ and $\angle B = 6(19) - 16 = 98^\circ$. Check: $82 + 98 = 180$.

$x = 19$; 82° and 98°

Problem 6 Diagonals with algebra

The diagonals bisect each other: $3x + 1 = 5x - 9$, so $10 = 2x$ and $x = 5$.

$AE = 16$, so $AC = 2(16) = 32$.

$x = 5$; $AC = 32$

Problem 7 Is it a parallelogram

- a. Both pairs of opposite sides are congruent: yes.
- b. Both pairs of opposite angles are congruent: yes.
- c. No. An isosceles trapezoid has this pattern and is not a parallelogram.
- d. Yes: diagonals that bisect each other prove a parallelogram.

(a) yes, (b) yes, (c) no, (d) yes

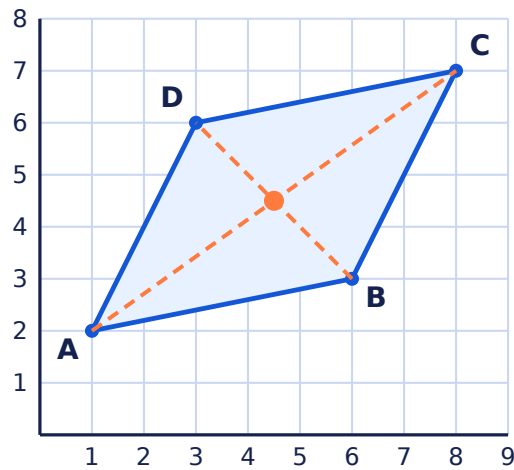
Problem 8 Make it a parallelogram

Both pairs of opposite sides must be congruent. First pair: $2x + 5 = 4x - 9$, so $14 = 2x$ and $x = 7$. Second pair: $3y - 1 = y + 7$, so $2y = 8$ and $y = 4$.

Check: $AB = CD = 19$ and $AD = BC = 11$.

$x = 7$, $y = 4$

Problem 9 Parallelogram on the plane



Both diagonals pass through (4.5, 4.5).

Midpoint of AC: $((1 + 8) \div 2, (2 + 7) \div 2) = (4.5, 4.5)$. Midpoint of BD: $((6 + 3) \div 2, (3 + 6) \div 2) = (4.5, 4.5)$.

The diagonals share a midpoint, so they bisect each other.

Yes, ABCD is a parallelogram

Problem 10 Rectangle diagonals

Rectangle diagonals are congruent: $5x - 3 = 3x + 11$, so $2x = 14$ and $x = 7$.

$AC = 5(7) - 3 = 32$, and $BD = 3(7) + 11 = 32$.

$x = 7$; both diagonals are 32

Problem 11 Diagonal of a rhombus

The four sides are equal: $52 \div 4 = 13$. The diagonals are perpendicular and bisect each other, so a right triangle has legs 5 (half of 10) and half of the other diagonal, with hypotenuse 13.

Half the other diagonal = $\sqrt{(13^2 - 5^2)} = \sqrt{144} = 12$, so the diagonal is 24. Area = $\frac{1}{2} \cdot 10 \cdot 24 = 120$.

Side 13; other diagonal 24; area 120

Problem 12 Angles of a rhombus

Consecutive angles are supplementary: $m\angle ABC = 180^\circ - 64^\circ = 116^\circ$.

Diagonals bisect the angles of a rhombus: $m\angle CAB = 64^\circ \div 2 = 32^\circ$ and $m\angle ABD = 116^\circ \div 2 =$

58°.

Check in the triangle formed by the diagonals: $32^\circ + 58^\circ + 90^\circ = 180^\circ$.

116°, 32° and 58°

Problem 13 Trapezoid midsegment

The midsegment is the average of the bases: $3x + 11 = \frac{1}{2}[(4x + 2) + (6x - 8)] = 5x - 3$. So $14 = 2x$ and $x = 7$.

The bases are $4(7) + 2 = 30$ and $6(7) - 8 = 34$, and the midsegment is $3(7) + 11 = 32$, the average of 30 and 34.

$x = 7$; bases 30 and 34; midsegment 32

Problem 14 Isosceles trapezoid

Base angles on the same base are congruent, so two angles are 72° . Angles between a leg and different bases are supplementary: $180^\circ - 72^\circ = 108^\circ$, so the other two angles are 108° .

Diagonals of an isosceles trapezoid are congruent: $2x + 9 = 4x - 7$, so $16 = 2x$ and $x = 8$. Each diagonal is 25.

Angles 72° , 72° , 108° , 108° ; diagonals 25

Problem 15 Building a kite

The crossbar is bisected by the spine, so each half is 12 inches. The top triangle has hypotenuse 15 and leg 12, so the top part of the spine is $\sqrt{15^2 - 12^2} = 9$. The bottom triangle has hypotenuse 20 and leg 12, so the bottom part is $\sqrt{20^2 - 12^2} = 16$.

Spine = $9 + 16 = 25$ inches. Area = $\frac{1}{2} \cdot 24 \cdot 25 = 300$ square inches.

Spine 25 in; area 300 in²

Problem 16 Garden bed fence

The opposite corner is 65° . The two other corners are supplementary to 65° : $180^\circ - 65^\circ = 115^\circ$.

Perimeter = $2(18 + 11.5) = 59$ feet. Cost = $59 \times 8.50 = \$501.50$.

Angles 65° , 115° , 115° ; perimeter 59 ft; cost \$501.50

Problem 17 Prove the diagonals bisect each other

Statement	Reason
1. $ABCD$ is a parallelogram	Given
2. $AB \parallel CD$ and $AB \cong CD$	Definition; opposite sides are congruent
3. $\angle BAE \cong \angle DCE$ and $\angle ABE \cong \angle CDE$	Alternate interior angles (parallel lines)
4. $\triangle ABE \cong \triangle CDE$	ASA
5. $AE \cong CE$ and $BE \cong DE$	CPCTC

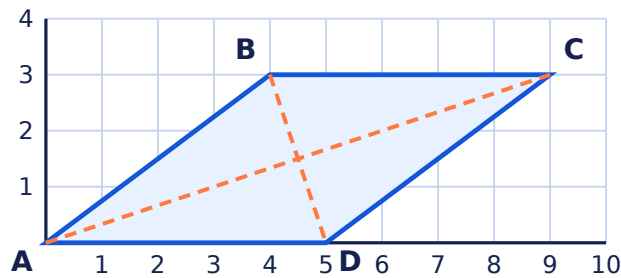
The diagonals bisect each other**Problem 18 Prove the converse**

Statement	Reason
1. $AB \cong CD$ and $AD \cong BC$	Given
2. $AC \cong AC$	Reflexive Property
3. $\triangle ABC \cong \triangle CDA$	SSS
4. $\angle BAC \cong \angle DCA$ and $\angle BCA \cong \angle DAC$	CPCTC
5. $AB \parallel CD$ and $BC \parallel AD$	Converse of the Alternate Interior Angles Theorem
6. $ABCD$ is a parallelogram	Definition of parallelogram

 $ABCD$ is a parallelogram**Problem 19 Prove rectangle diagonals congruent**

Statement	Reason
1. $PQRS$ is a rectangle	Given
2. $PQ \cong SR$	A rectangle is a parallelogram, so opposite sides are congruent
3. $QR \cong RQ$	Reflexive Property
4. $\angle PQR \cong \angle SRQ$	Both are right angles
5. $\triangle PQR \cong \triangle SRQ$	SAS
6. $PR \cong SQ$	CPCTC

 $PR \cong QS$ **Problem 20 Coordinate proof of a rhombus**



Rhombus ABCD with A at 0 comma 0, B at 4 comma 3, C at 9 comma 3 and D at 5 comma 0, with both diagonals dashed

Side lengths: $AB = \sqrt{4^2 + 3^2} = 5$, $BC = 9 - 4 = 5$, $CD = \sqrt{(9 - 5)^2 + 3^2} = 5$, $DA = 5 - 0 = 5$. All four sides are equal, so the quadrilateral is a rhombus.

Slope of $AC = 3 \div 9 = 1/3$. Slope of $BD = (0 - 3) \div (5 - 4) = -3$. The product $(1/3)(-3) = -1$, so the diagonals are perpendicular.

Rhombus with all sides 5; diagonals perpendicular (slopes 1/3 and -3)