

Points, Segments and Angles: Core Definitions

Grade 10 – Geometry math · Lesson · Standards: G-CO.A.1

Quick answer: Geometry builds on three undefined terms: point, line and plane. From them come segments, rays and angles. The Segment Addition and Angle Addition Postulates say that parts add up to the whole, while midpoints and bisectors split a figure into two equal parts. Special angle pairs, such as linear pairs and vertical angles, let algebra find unknown measures.

Standards: G-CO.A.1 · **Level:** Grade 10 Geometry

What are the undefined terms of geometry?

Every definition rests on earlier words, so geometry starts with three ideas accepted without a formal definition.

Undefined terms

- **Point:** a location with no size, named by a capital letter such as K .
- **Line:** a straight path with no thickness that never ends in either direction.
- **Plane:** a flat surface with no thickness that extends forever in all directions.

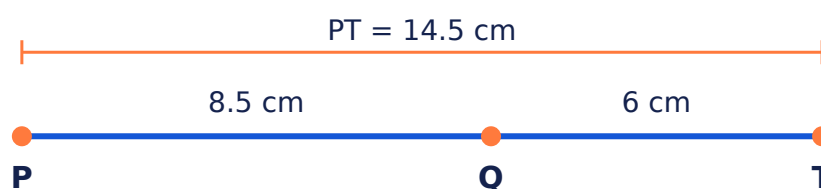
From these terms we define a **segment** (two endpoints and every point between them), a **ray** (one endpoint and all points in one direction) and an **angle** (two rays sharing an endpoint called the vertex). **Parallel lines** lie in one plane and never meet. **Perpendicular lines** meet at a right angle.

How do you add segments and angles?

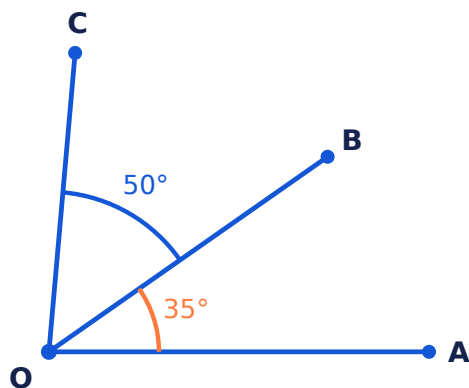
Two addition postulates

If point Q lies between P and T , then $PQ + QT = PT$.

If ray OB lies inside $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$.



Segment Addition: $PQ + QT = PT$.



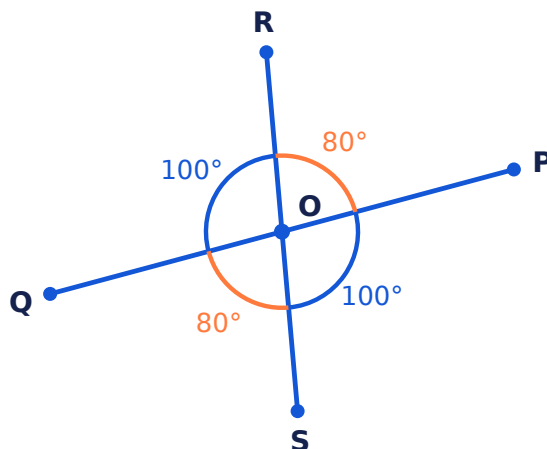
Angle Addition: $\angle AOB + \angle BOC = \angle AOC = 85^\circ$.

What do midpoints and bisectors do?

A **midpoint** splits a segment into two congruent segments. A **bisector** of a segment or an angle is a line, ray or segment that cuts it into two congruent parts. If M is the midpoint of segment PT , then $PM = MT = \frac{1}{2}PT$.

Which angle pairs have a fixed relationship?

Pair	Description	Relationship
Complementary	Two angles whose measures add to 90°	$x + y = 90^\circ$
Supplementary	Two angles whose measures add to 180°	$x + y = 180^\circ$
Linear pair	Adjacent angles whose outer rays form a line	Supplementary
Vertical angles	Opposite angles formed by two crossing lines	Equal measures



Vertical angles are equal (80° and 80°). Neighbors form linear pairs ($80^\circ + 100^\circ = 180^\circ$).

How do you solve an angle or segment problem with algebra?

Method

1. Sketch the figure and mark what is given.
2. Choose the rule: addition postulate, midpoint, bisector, linear pair or vertical angles.
3. Write an equation using that rule.
4. Solve for x , then substitute to find every requested measure.

5. Check that the pieces add up to the whole.

Worked example

Ray OB bisects $\angle AOC$, with $m\angle AOB = (5x - 4)^\circ$ and $m\angle BOC = (3x + 12)^\circ$. A bisector makes equal halves, so $5x - 4 = 3x + 12$, which gives $2x = 16$ and $x = 8$. Each half measures 36° , so $m\angle AOC = 72^\circ$.

Common mistake

Vertical angles are equal, never supplementary. A linear pair adds to 180° , but two angles that merely look like they form a line must be shown to share a ray first.

Mathbot's tip

Draw the picture before touching the algebra. Label the whole, label the parts, and the equation almost writes itself.

Key points to remember

- Point, line and plane are undefined terms; every other geometric word is defined from them.
- If Q is between P and T , then $PQ + QT = PT$.
- Complementary angles total 90° and supplementary angles total 180° .
- A linear pair is supplementary; vertical angles are equal.
- A midpoint or bisector creates two congruent halves.

Points, Segments and Angles: Core Definitions: frequently asked questions

Why does geometry have undefined terms?

Any definition uses other words, so the chain must start somewhere. Point, line and plane are described by examples and then used to define everything else.

What is the difference between a segment, a ray and a line?

A segment has two endpoints and a measurable length. A ray has one endpoint and goes on forever one way. A line has no endpoints and goes on forever both ways.

Are vertical angles the same as a linear pair?

No. Vertical angles are opposite each other and are equal. A linear pair is two neighboring angles on a line, and they add to 180° .

What does it mean when a ray bisects an angle?

The ray starts at the vertex and splits the angle into two angles with the same measure. Each half is exactly one half of the whole angle.

Can two angles be both complementary and supplementary?

Not with positive measures. Complementary angles add to 90° and supplementary angles add to 180° , so no pair of angles can satisfy both.