

# Points, Segments and Angles: Core Definitions

## Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-CO.A.1

Every solution names the rule used, writes the equation, solves it and checks the result against the figure. Compare each line of work with the final bold answer, and redo any problem where the equation differs.

**Standards:** G-CO.A.1 · **Level:** Grade 10 Geometry

### Problem 1 Adding two segments

$$PR = PQ + QR = 14.5 + 9.25 = 23.75.$$

**PR = 23.75 inches**

### Problem 2 Finding a midpoint length

A midpoint creates congruent halves, so  $MS = RM = 18.6$ . Then  $RS = 18.6 + 18.6 = 37.2$ .

**MS = 18.6 cm and RS = 37.2 cm**

### Problem 3 Complement and supplement

Complement:  $90^\circ - 37^\circ = 53^\circ$ . Supplement:  $180^\circ - 37^\circ = 143^\circ$ .

**Complement  $53^\circ$ , supplement  $143^\circ$**

### Problem 4 Angles in a linear pair

A linear pair is supplementary:  $m\angle BOC = 180^\circ - 118^\circ = 62^\circ$ .

**$62^\circ$**

### Problem 5 Vertical angles

The vertical angle  $\angle QOS$  is equal to  $\angle POR$ , so it measures  $74^\circ$ . The other two angles each form a linear pair with a  $74^\circ$  angle:  $180^\circ - 74^\circ = 106^\circ$ .

**$74^\circ$ ,  $106^\circ$  and  $106^\circ$**

### Problem 6 Segment addition with algebra

Segment Addition:  $(3x + 2) + (5x - 7) = 51$ , so  $8x - 5 = 51$ ,  $8x = 56$  and  $x = 7$ .

$AB = 3(7) + 2 = 23$  and  $BC = 5(7) - 7 = 28$ . Check:  $23 + 28 = 51$ .

**$x = 7$ ,  $AB = 23$ ,  $BC = 28$**

### Problem 7 Angle addition with algebra

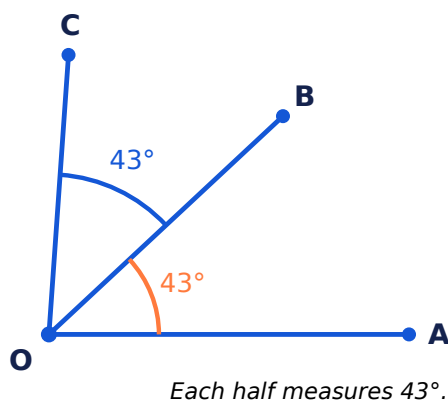
$(2x + 10) + (3x - 8) = 112$  gives  $5x + 2 = 112$ , so  $x = 22$ .

$m\angle AOB = 2(22) + 10 = 54^\circ$  and  $m\angle BOC = 3(22) - 8 = 58^\circ$ . Check:  $54 + 58 = 112$ .

**$x = 22$ ,  $54^\circ$  and  $58^\circ$**

### Problem 8 Angle bisector equation

A bisector gives equal halves:  $4x + 3 = 6x - 17$ , so  $20 = 2x$  and  $x = 10$ .



Each half is  $4(10) + 3 = 43^\circ$ , so  $m\angle AOC = 2 \cdot 43^\circ = 86^\circ$ .

**$x = 10$  and  $m\angle AOC = 86^\circ$**

### Problem 9 Midpoint on a number line

Midpoint:  $(-7 + 12) \div 2 = 5 \div 2 = 2.5$ . Length:  $12 - (-7) = 19$ .

**Midpoint at 2.5; DE = 19**

### Problem 10 A supplement and a complement

Let the angle be  $x$ . Then  $180 - x = 3(90 - x) + 25 = 295 - 3x$ , so  $2x = 115$  and  $x = 57.5$ .

Check: complement  $32.5$ , and  $3(32.5) + 25 = 122.5$ , which is the supplement  $180 - 57.5$ .

**$57.5^\circ$**

### **Problem 11 Vertical angles with algebra**

Vertical angles are equal:  $5x - 12 = 3x + 22$ , so  $2x = 34$  and  $x = 17$ .

Each vertical angle is  $5(17) - 12 = 73^\circ$ . The neighbors measure  $180^\circ - 73^\circ = 107^\circ$ .

**$x = 17$ ;  $73^\circ$  and  $107^\circ$**

### **Problem 12 Linear pair with algebra**

$(7x + 9) + (5x + 15) = 180$  gives  $12x + 24 = 180$ , so  $12x = 156$  and  $x = 13$ .

The angles are  $7(13) + 9 = 100^\circ$  and  $5(13) + 15 = 80^\circ$ , and  $100 + 80 = 180$ .

**$100^\circ$  and  $80^\circ$**

### **Problem 13 A right angle split in two**

Perpendicular lines form a  $90^\circ$  angle, so  $(2x + 7) + (3x - 2) = 90$ . Then  $5x + 5 = 90$  and  $x = 17$ .

$\angle ABC = 2(17) + 7 = 41^\circ$  and  $\angle CBD = 3(17) - 2 = 49^\circ$ .

**$41^\circ$  and  $49^\circ$**

### **Problem 14 Always, sometimes or never**

- a. Always: parallel lines are defined as coplanar lines that never meet.
- b. Sometimes: three points can lie on one line, but most triples do not.
- c. Never: a ray has no second endpoint, so no point is halfway to anything.
- d. Sometimes: vertical angles are equal, so they add to  $180^\circ$  only when each measures  $90^\circ$ .

**(a) always, (b) sometimes, (c) never, (d) sometimes**

### **Problem 15 Hiking trail midpoint**

$TC = 2.4 + 1.75 = 4.15$  miles. The midpoint is  $4.15 \div 2 = 2.075$  miles from  $T$ .

The lookout is  $2.4$  miles from  $T$ , and  $2.4 - 2.075 = 0.325$ , so the fountain is before the lookout, on the trailhead side.

**TC = 4.15 mi; the fountain is 0.325 mi from the lookout, toward the trailhead**

### Problem 16 Robot sensor beam

The bisector makes  $m\angle AOB = 128^\circ \div 2 = 64^\circ$ , and also  $m\angle BOC = 64^\circ$ .

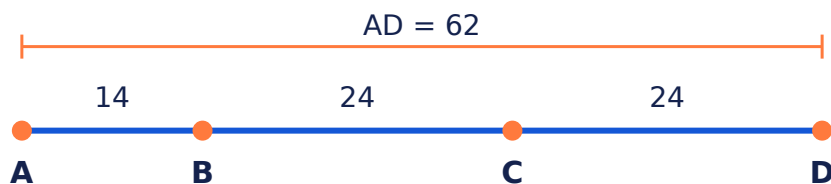
Ray  $OD$  lies inside  $\angle BOC$ , so by Angle Addition  $m\angle DOC = 64^\circ - 19^\circ = 45^\circ$ .

**$m\angle AOB = 64^\circ$  and  $m\angle DOC = 45^\circ$**

### Problem 17 Four collinear points

$x + (2x - 4) + (x + 10) = 62$  gives  $4x + 6 = 62$ , so  $x = 14$ .

$AB = 14$ ,  $BC = 24$  and  $CD = 24$ . The midpoint  $M$  is 31 from  $A$ . Since  $B$  is 14 from  $A$  and  $C$  is 38 from  $A$ , and  $14 < 31 < 38$ ,  $M$  is between  $B$  and  $C$ , 17 units from  $B$ .



*M is 31 from A, between B and C.*

**$x = 14$ ;  $AB = 14$ ,  $BC = 24$ ,  $CD = 24$ ; yes,  $M$  is between  $B$  and  $C$**

### Problem 18 Linear pair, vertical angles, bisector

$(3x + 5) + (2x + 15) = 180$  gives  $5x + 20 = 180$  and  $x = 32$ .

$\angle 1 = 3(32) + 5 = 101^\circ$  and  $\angle 2 = 2(32) + 15 = 79^\circ$ . Vertical angles are equal, so  $\angle 3 = 101^\circ$  and  $\angle 4 = 79^\circ$ . A bisector of  $\angle 2$  makes two angles of  $79^\circ \div 2 = 39.5^\circ$ .

**$101^\circ$ ,  $79^\circ$ ,  $101^\circ$ ,  $79^\circ$ ; bisected pieces  $39.5^\circ$**

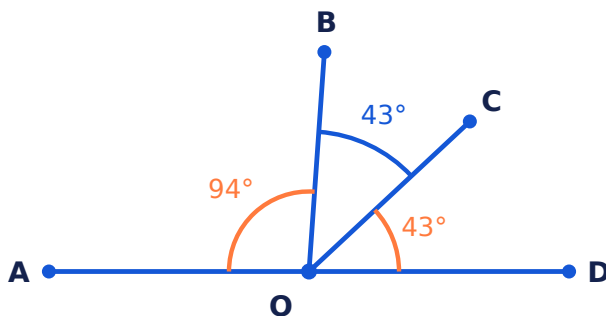
### Problem 19 Highway mile markers

Rest stop:  $(14.6 + 31.2) \div 2 = 45.8 \div 2 = 22.9$ . The gas station is at  $22.9 + 3.5 = 26.4$ . Its distance to Exit 2 is  $31.2 - 26.4 = 4.8$ .

**Rest stop at marker 22.9; gas station is 4.8 miles from Exit 2**

### Problem 20 Bisector on a straight angle

$\angle COD = \angle BOC = (x + 18)^\circ$ . The three angles fill a straight angle:  $(4x - 6) + 2(x + 18) = 180$ , so  $6x + 30 = 180$  and  $x = 25$ .



*The three angles fill  $180^\circ$ .*

$\angle AOB = 94^\circ$ ,  $\angle BOC = 43^\circ$  and  $\angle COD = 43^\circ$ . Check:  $94 + 43 + 43 = 180$ .

**$x = 25$ ;  $94^\circ$ ,  $43^\circ$ ,  $43^\circ$**