

Probability Rules, Conditioning and Fair Decisions

Grade 10 – Geometry math · Lesson · Standards: S-CP.A.1, S-CP.A.2, S-CP.A.3, S-CP.A.4, S-CP.A.5, S-CP.B.6, S-CP.B.7, S-CP.B.8, S-CP.B.9, S-MD.B.6, S-MD.B.7

Quick answer: Probability rules combine events: subtract the overlap when adding probabilities, and divide by the condition when finding a conditional probability. Events are independent when $P(A \text{ and } B)$ equals $P(A) \cdot P(B)$. Counting with permutations and combinations and using tree diagrams support fair decisions.

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What is a sample space and how are events combined?

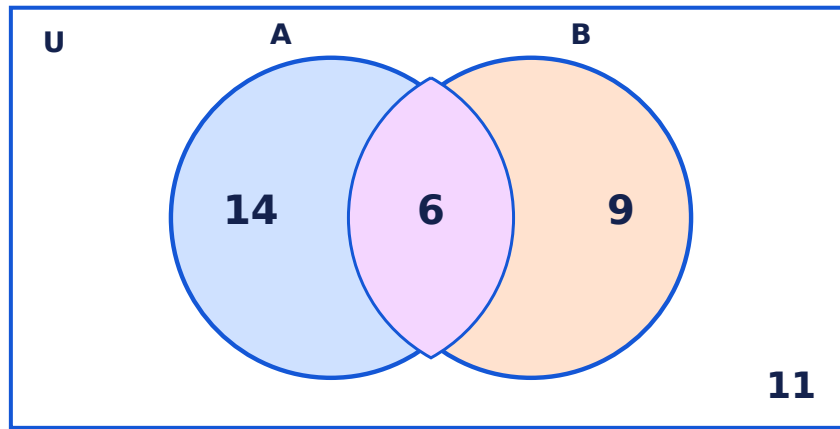
List every outcome that can happen and you have the sample space; any chosen part of it is an event. Writing $A \cup B$ asks for outcomes in at least one event, $A \cap B$ asks for outcomes in both, and A' collects everything outside A .

Die 1	Die 2					
	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Sample space for two dice: $P(\text{sum is } 7) = 6/36 = 1/6$.

Core rules

- Complement: $P(A') = 1 - P(A)$.
- Addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- Conditional probability: $P(A | B) = \frac{P(A \cap B)}{P(B)}$.
- Multiplication rule: $P(A \cap B) = P(A) \cdot P(B | A)$.



Counts in a Venn diagram: $P(A \cup B) = 29/40$.

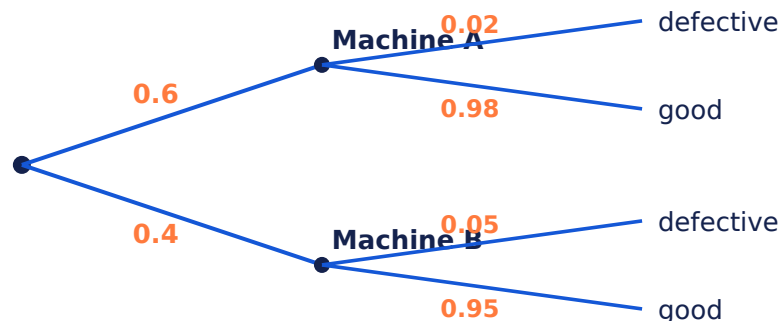
When are two events independent?

Independent events

Two events are independent when learning that one happened leaves the chance of the other unchanged. In symbols, $P(A \cap B) = P(A) \cdot P(B)$, equivalently $P(A | B) = P(A)$.

How do counting methods and tree diagrams help?

Ordered selections use permutations, ${}_nP_r = n!/(n - r)!$. Unordered groups use combinations, ${}_nC_r = n!/(r!(n - r)!)$. A tree diagram multiplies along branches and adds across branches.



$$P(\text{defective}) = 0.6(0.02) + 0.4(0.05) = 0.032.$$

Method

1. Write down the sample space or fill in the table of counts.
2. Label the events and decide if the question wants a union, an intersection or a conditional.
3. Pick the matching rule and calculate.
4. Confirm each probability lies between 0 and 1.

Example

Of 120 riders, 45 take the train and 30 of those also bike. Then $P(\text{bike} \mid \text{train}) = 30/45 = 2/3$, while $P(\text{train and bike}) = 30/120 = 1/4$.

Common mistake

Mixing up $P(A \mid B)$ and $P(B \mid A)$. The denominator is the total of the condition, the event after the bar.

Mathbot's tip For a fair decision with a random number generator, give every choice the same number of outcomes. Unequal shares mean an unfair method.

Key points to remember

- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$; $P(\text{not } A) = 1 - P(A)$.
- $P(A \mid B) = P(A \text{ and } B) \div P(B)$; $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.
- Independent events satisfy $P(A \text{ and } B) = P(A) \cdot P(B)$.
- ${}_nP_r$ counts ordered selections and ${}_nC_r$ counts groups where order does not matter.

Probability Rules, Conditioning and Fair Decisions: frequently asked questions

Why do we subtract $P(A \text{ and } B)$ in the addition rule?

Outcomes in both events are counted twice when you add $P(A)$ and $P(B)$. Subtracting the overlap once corrects the double count.

How do I test whether two events are independent?

Compute $P(A) \cdot P(B)$ and compare it with $P(A \text{ and } B)$. If they are equal, the events are independent. You can also check whether $P(A \mid B)$ equals $P(A)$.

What is the difference between a permutation and a combination?

A permutation counts arrangements where order matters, such as gold, silver and bronze. A combination counts groups where order does not matter, such as a committee.

How do two-way tables give conditional probabilities?

Restrict attention to one row or column, the condition, and divide the cell count by that row or column total instead of the grand total.

What makes a decision method fair?

Every person or option must have the same chance, such as a random draw or a random number generator that assigns equal numbers of outcomes to each choice.