

Solving Right Triangles with Sine, Cosine and Tangent

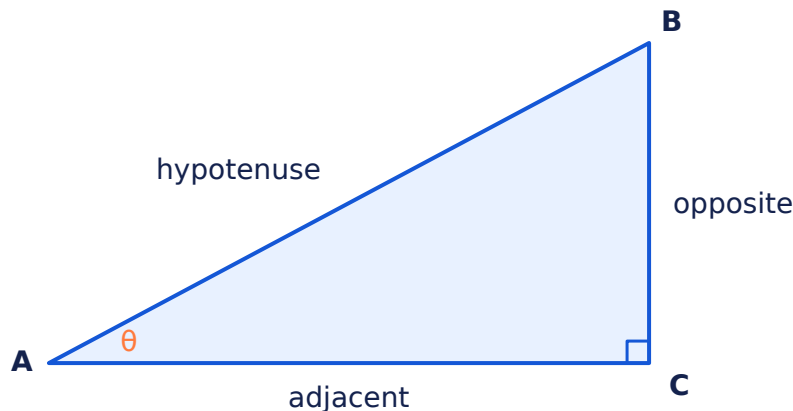
Grade 10 – Geometry math · Lesson · Standards: G-SRT.C.6, G-SRT.C.7, G-SRT.C.8

Quick answer: Sine, cosine and tangent are ratios of side lengths in a right triangle, fixed by one acute angle: sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, tangent is opposite over adjacent. With them you can find missing sides and angles.

Standards: G-SRT.C.6, G-SRT.C.7, G-SRT.C.8 · **Level:** Grade 10 Geometry

What do sine, cosine and tangent measure?

Two right triangles sharing one acute angle are similar, which means a given pair of sides always forms the same quotient. Mathematicians named those quotients.



Sides named from the point of view of angle A.

Trigonometric ratios

Take any acute angle θ of a right triangle. Then

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

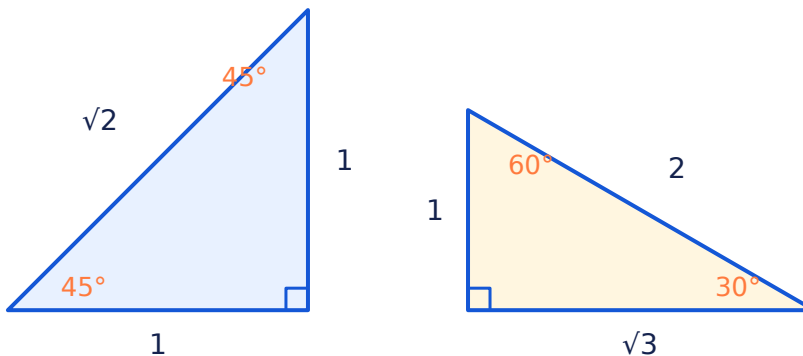
Many students remember them as SOH CAH TOA.

How are the two acute angles connected?

Complementary angles

The opposite side of one acute angle is the adjacent side of the other. So $\sin \theta = \cos(90^\circ - \theta)$ and $\cos \theta = \sin(90^\circ - \theta)$.

Which triangles have exact ratios?



Special right triangles: side ratios $1 : 1 : \sqrt{2}$ and $1 : \sqrt{3} : 2$.

Angle	sin	cos	tan
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

How do you solve a right triangle?

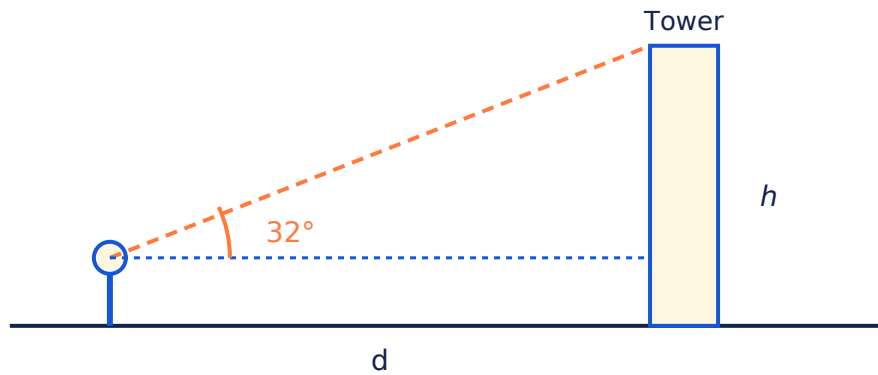
Method

1. Draw the figure and write the given measures on it.
2. Choose a reference angle, then tag each side as opposite, adjacent or hypotenuse.
3. Select the ratio holding exactly one unknown and two known values.
4. Solve the resulting equation and confirm the hypotenuse stays the longest side.
5. When an angle is missing, press the inverse function ($\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$).

Example

A drone hovers directly over a point 40 yards from a flagpole base, and its camera looks down at the pole base at an angle of depression of 52° . How high is it?

The angle of depression equals the angle of elevation from the pole base, so the 40-yard side is adjacent and the height is opposite: $\tan 52^\circ = \frac{x}{40}$. Then $x = 40 \cdot \tan 52^\circ \approx 51.2$ yards.



Angle of elevation: measured up from the horizontal.

Common mistake

Calculator in the wrong mode. Check that it is set to degrees before using sin, cos or tan.

Mathbot's tip Label the three sides from the angle you are using. Ratios of the other angle swap opposite and adjacent.

Key points to remember

- $\sin \theta = \text{opposite} \div \text{hypotenuse}$, $\cos \theta = \text{adjacent} \div \text{hypotenuse}$, $\tan \theta = \text{opposite} \div \text{adjacent}$.
- $\sin \theta = \cos(90^\circ - \theta)$ for any acute angle θ .
- A 45-45-90 triangle has sides in ratio 1 : 1 : $\sqrt{2}$; a 30-60-90 triangle has 1 : $\sqrt{3}$: 2.
- Angles of elevation and depression are measured from the horizontal line.

Solving Right Triangles with Sine, Cosine and Tangent: frequently asked questions

Why do the trigonometric ratios depend only on the angle?

Right triangles that share an acute angle are similar by AA, so their corresponding sides are proportional. Any ratio of two sides is therefore the same in all of them.

When should I use sine, cosine or tangent?

Look at the unknown and the known side. Hypotenuse and opposite use sine, hypotenuse and adjacent use cosine, and the two legs use tangent.

What is the difference between an angle of elevation and depression?

Elevation is measured upward from a horizontal line to the line of sight. Depression is measured downward from the horizontal. The two are equal alternate interior angles.

How do I find an angle when I know two sides?

Write the ratio of the two sides and apply the inverse function. For example, if $\tan \theta = 5/9$, then $\theta = \tan^{-1}(5/9) \approx 29.1^\circ$.

Why is $\sin 30^\circ$ exactly $1/2$?

Two 30-60-90 triangles joined along the long leg form an equilateral triangle. The short leg is half the hypotenuse, so the opposite-over-hypotenuse ratio for 30° is $1/2$.