

Translations, Reflections and Rotations Explained

Grade 10 – Geometry math · Lesson · Standards: G-CO.A.2, G-CO.A.3, G-CO.A.4, G-CO.A.5

Quick answer: A rigid transformation relocates a figure while keeping all lengths and angle measures. Translations, reflections and rotations qualify, so each image is congruent to its original. On the coordinate plane each motion has a rule, such as $(x, y) \rightarrow (y, x)$ for a reflection over $y = x$. Dilations are not rigid because they change size.

Standards: G-CO.A.2, G-CO.A.3, G-CO.A.4, G-CO.A.5 · **Level:** Grade 10 Geometry

How does geometry treat motion as a function?

Every **transformation** is a function: each input point, the **preimage**, is sent to exactly one output point, the **image**. We write the image of point A as A' . A **rigid motion** keeps every distance and every angle, which makes the image congruent to the preimage.

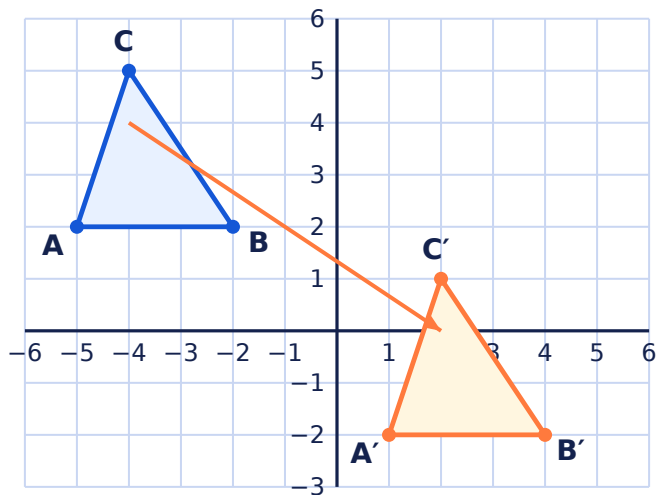
The three rigid motions

- **Translation:** shifts all points equally along one vector, so the figure slides without turning.
- **Reflection:** flips the plane across a mirror line. That line cuts each segment from a point to its image into two equal halves at a right angle.
- **Rotation:** spins the plane through a chosen angle around a fixed center point.

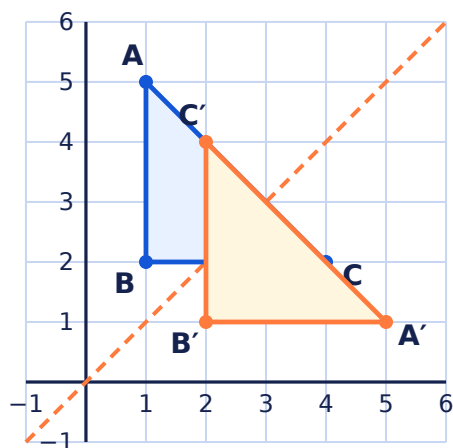
A dilation is *not* rigid: it changes lengths, so it does not preserve distance.

What are the coordinate rules for each motion?

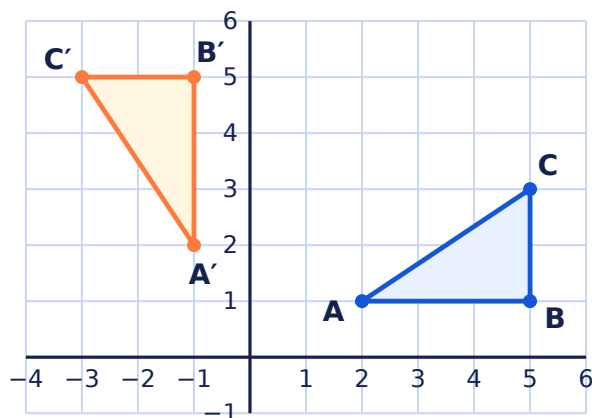
Transformation	Rule on the coordinate plane
Translation by (a, b)	$(x, y) \rightarrow (x + a, y + b)$
Reflection over the x -axis	$(x, y) \rightarrow (x, -y)$
Reflection over the y -axis	$(x, y) \rightarrow (-x, y)$
Reflection over $y = x$	$(x, y) \rightarrow (y, x)$
Quarter-turn counterclockwise about the origin	$(x, y) \rightarrow (-y, x)$
Half-turn about the origin	$(x, y) \rightarrow (-x, -y)$
Three-quarter turn counterclockwise about the origin	$(x, y) \rightarrow (y, -x)$



Translation by vector $(6, -4)$: $(x, y) \rightarrow (x + 6, y - 4)$.



Reflection over $y = x$ swaps the coordinates: $(x, y) \rightarrow (y, x)$.



A 90° counterclockwise rotation about the origin: $(x, y) \rightarrow (-y, x)$.

When does a figure have symmetry?

A rigid motion that carries a figure onto itself is a **symmetry**. A non-square rectangle can be

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flipped over two mirror lines and turned a half-turn, and it lands on itself each time. A regular n -gon has n lines of symmetry and rotational symmetry of $360^\circ \div n$, so a regular hexagon maps onto itself after a 60° turn.

How do you follow two or more motions in a row?

Method

1. Apply the first motion to every vertex and record the new coordinates.
2. Apply the second motion to the new points.
3. Check that side lengths in the final image match the preimage.
4. To undo a sequence, reverse the order and invert each step.

Worked example

Reflect $P(3, -2)$ over the y -axis, then translate by $\langle 0, 5 \rangle$. The reflection gives $(-3, -2)$. The translation adds 5 to the y -coordinate: **$(-3, 3)$** . Translating first and reflecting second gives $(3, 3)$ and then $(-3, 3)$, the same point here. For most pairs of motions, however, *order matters*.

Common mistake

A rotation of 90° clockwise equals 270° counterclockwise, and its rule is $(x, y) \rightarrow (y, -x)$. Mixing up the direction swaps the signs.

Mathbot's tip

Test every rule on one easy point such as $(1, 0)$. If the image lands where the picture says, the rule is right.

Key points to remember

- Each transformation assigns one image point to every point of the plane.
- Translations, reflections and rotations preserve distance and angle measure.
- A reflection over $y = x$ swaps coordinates; a 90° counterclockwise rotation sends (x, y) to $(-y, x)$.
- A regular n -gon has n lines of symmetry and rotational symmetry of $360^\circ \div n$.
- The order of a sequence of transformations can change the final image.

Translations, Reflections and Rotations Explained: frequently asked questions

What makes a transformation rigid?

It preserves the distance between every pair of points and all angle measures. Translations, reflections and rotations satisfy this, so the image is congruent to the preimage.

How do you find the line of reflection?

Join a point to its image. The line of reflection is the perpendicular bisector of that segment, so it passes through the midpoint at a right angle.

What is the rule for a 90° clockwise rotation about the origin?

A 90° clockwise turn equals a 270° counterclockwise turn, so (x, y) maps to $(y, -x)$. For example, $(2, 5)$ maps to $(5, -2)$.

Does the order of two transformations matter?

Often it does. Take the point $(2, 3)$, a translation by $(1, 4)$ and a reflection over the x-axis. Translating first gives $(3, 7)$ and then $(3, -7)$. Reflecting first gives $(2, -3)$ and then $(3, 1)$. Different results, so always test with a point.

How many symmetries does a square have?

A square has four lines of reflection symmetry and rotational symmetry of 90° , 180° and 270° . A rectangle that is not a square has only two lines and a 180° rotation.