

Translations, Reflections and Rotations Explained

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-CO.A.2, G-CO.A.3, G-CO.A.4, G-CO.A.5

Each solution applies a coordinate rule, shows the image points and checks a length or a picture when helpful. Compare the order in which motions are applied, because the final image can depend on it.

Standards: G-CO.A.2, G-CO.A.3, G-CO.A.4, G-CO.A.5 · **Level:** Grade 10 Geometry

Problem 1 Translate a point

Subtract 5 from the x-coordinate and add 4 to the y-coordinate: $(3 - 5, -2 + 4) = (-2, 2)$.

P'(-2, 2)

Problem 2 Reflect over an axis

Over the x-axis the sign of y changes: $(-4, -6)$. Over the y-axis the sign of x changes: $(4, 6)$.

(a) (-4, -6), (b) (4, 6)

Problem 3 Half-turn about the origin

A 180° rotation changes both signs: $(-7, 3)$.

R'(-7, 3)

Problem 4 Quarter-turn rotation

Use $(x, y) \rightarrow (-y, x)$: $(-(-1), 4) = (1, 4)$.

S'(1, 4)

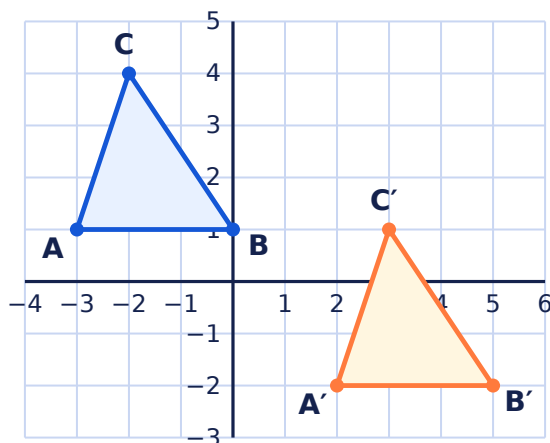
Problem 5 Reflect over $y = x$

Reflecting over $y = x$ swaps the coordinates: $(5, -2)$.

T'(5, -2)

Problem 6 Describe a translation

The vector goes from A to A' : $\langle 2 - (-3), -2 - 1 \rangle = \langle 5, -3 \rangle$. Apply it to the other vertices: $B' = (0 + 5, 1 - 3) = (5, -2)$ and $C' = (-2 + 5, 4 - 3) = (3, 1)$.



Triangle and its translated image.

Vector $\langle 5, -3 \rangle$; $B'(5, -2)$; $C'(3, 1)$

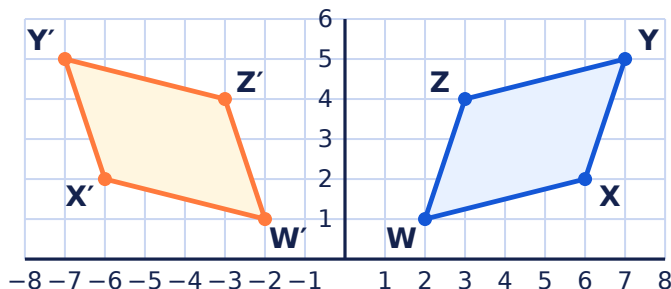
Problem 7 Rigid or not rigid

- a. The segment goes to $(4, -1)-(6, -1)$, still length 2: rigid (a translation).
- b. The segment goes to $(0, 0)-(4, 0)$, length 4: not rigid (a dilation).
- c. The segment is unchanged in length, and the rule is a reflection: rigid.
- d. The segment goes to $(0, 0)-(6, 0)$, length 6: not rigid (a horizontal stretch).

(a) rigid, (b) not rigid, (c) rigid, (d) not rigid

Problem 8 Reflect a quadrilateral

Negate each x -coordinate: $W'(-2, 1)$, $X'(-6, 2)$, $Y'(-7, 5)$, $Z'(-3, 4)$.

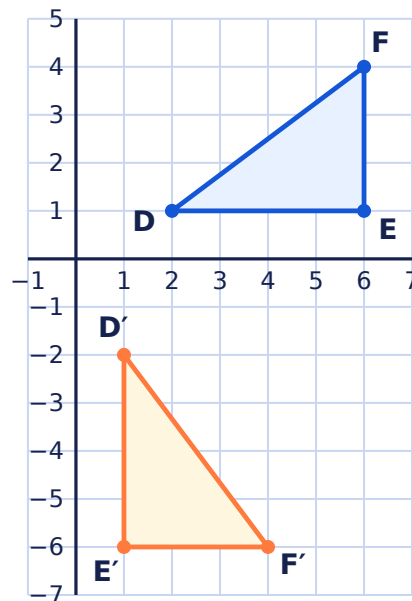


Preimage and image are mirror images across the y -axis.

$W'(-2, 1)$, $X'(-6, 2)$, $Y'(-7, 5)$, $Z'(-3, 4)$

Problem 9 Clockwise rotation

A 90° clockwise rotation is a 270° counterclockwise rotation: $(x, y) \rightarrow (y, -x)$. $D'(1, -2)$, $E'(1, -6)$, $F'(4, -6)$.



Rotated image in the fourth quadrant.

$D'(1, -2)$, $E'(1, -6)$, $F'(4, -6)$

Problem 10 Reflect over $y = -x$

For P : $(-(-5), -3) = (5, -3)$. For Q : $(-(-6), -(-2)) = (6, 2)$.

$P'(5, -3)$, $Q'(6, 2)$

Problem 11 Symmetries of a rectangle

Reflections: over the x -axis and over the y -axis, so two lines of symmetry. The diagonals are not lines of symmetry because the sides are unequal.

Rotations: only 180° about the origin, since a 90° turn would swap the 6-unit and 4-unit sides.

A square has four lines of symmetry (two axes, two diagonals) and rotations of 90° , 180° and 270° .

Rectangle: 2 lines, 180° rotation; square: 4 lines, rotations of 90° , 180° , 270°

Problem 12 Regular polygon symmetry

A regular n -gon has n lines of symmetry and a smallest rotation of $360^\circ \div n$.

Polygon	Lines of symmetry	Smallest rotation
Pentagon	5	$360 \div 5 = 72^\circ$
Octagon	8	$360 \div 8 = 45^\circ$
12-gon	12	$360 \div 12 = 30^\circ$

Pentagon 5 lines, 72° ; octagon 8 lines, 45° ; 12-gon 12 lines, 30°

Problem 13 Order of two motions

Translate first: $(1, -5)$; then reflect over the x -axis: $(1, 5)$.

Reflect first: $(-7, -4)$; then translate: $(1, -13)$.

The two final points differ, so the order matters.

$(1, 5)$ versus $(1, -13)$

Problem 14 Find the line of reflection

The line of reflection is the perpendicular bisector of AA' . The points differ only in y , so the line is horizontal through the midpoint: $y = (5 + (-1)) \div 2 = 2$.

Reflecting over $y = 2$ keeps x and sends y to $2(2) - y$. For B : $4 - (-3) = 7$, so $B' = (6, 7)$.

Line $y = 2$; $B'(6, 7)$

Problem 15 Robot docking route

After the translation: $(-3 + 7, 2 - 4) = (4, -2)$. Reflecting over the x -axis gives $(4, 2)$.

The first move has length $\sqrt{7^2 + 4^2} = \sqrt{65} \approx 8.1$ feet.

Dock at $(4, 2)$; first move ≈ 8.1 ft

Problem 16 Rotation about another center

Move the center to the origin by subtracting $(2, 1)$: P becomes $(3, 2)$.

90° counterclockwise: $(3, 2) \rightarrow (-2, 3)$. Add $(2, 1)$ back: $(0, 4)$.

180° : $(3, 2) \rightarrow (-3, -2)$. Add $(2, 1)$ back: $(-1, -1)$.

Check: $CP = \sqrt{(9 + 4)} = \sqrt{13}$, and the distance from C to $(0, 4)$ is $\sqrt{(4 + 9)} = \sqrt{13}$.

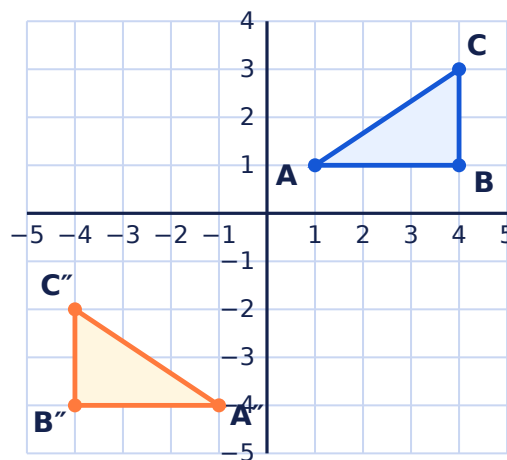
$(0, 4)$ and $(-1, -1)$

Problem 17 Reflection then translation

Reflection over the y -axis: $A'(-1, 1)$, $B'(-4, 1)$, $C'(-4, 3)$.

Translation by $\langle 0, -5 \rangle$: $A''(-1, -4)$, $B''(-4, -4)$, $C''(-4, -2)$.

Check: $AB = 4 - 1 = 3$ and $A''B'' = |-4 - (-1)| = 3$.



Final image after both motions.

$A''(-1, -4)$, $B''(-4, -4)$, $C''(-4, -2)$; both lengths are 3

Problem 18 Identify the rotation

Test the rule $(x, y) \rightarrow (-y, x)$: $P(1, 2) \rightarrow (-2, 1) \checkmark$, $Q(4, 2) \rightarrow (-2, 4) \checkmark$, $R(4, 4) \rightarrow (-4, 4) \checkmark$.

The horizontal side PQ of length 3 became the vertical side $P'Q'$ of length 3, which fits a quarter turn.

A 90° counterclockwise rotation about the origin: $(x, y) \rightarrow (-y, x)$

Problem 19 Mosaic with four-fold symmetry

Apply $(x, y) \rightarrow (-y, x)$ repeatedly: $(3, 1) \rightarrow (-1, 3) \rightarrow (-3, -1) \rightarrow (1, -3)$, and one more turn returns to $(3, 1)$.

The reflection over $y = x$ swaps coordinates, so it maps $(3, 1)$ to $(1, 3)$.

$(3, 1), (-1, 3), (-3, -1), (1, -3)$; the line $y = x$

Problem 20 Two reflections make a translation

Reflection over $x = 1$ sends (x, y) to $(2 - x, y)$. Over $x = 4$ it sends (x, y) to $(8 - x, y)$.

For P : $(0, 2) \rightarrow (2, 2) \rightarrow (6, 2)$. The motion is the translation $\langle 6, 0 \rangle$, which is twice the distance 3 between the lines.

General point: $(x, y) \rightarrow (2 - x, y) \rightarrow (8 - (2 - x), y) = (x + 6, y)$.

Final point $(6, 2)$; the translation $\langle 6, 0 \rangle$