

Similar Triangles, Proportions and Indirect Measurement

Grade 10 – Geometry math · Lesson · Standards: G-SRT.B.4, G-SRT.B.5

Quick answer: Triangles are similar when two angles match (AA), when two sides are proportional with equal included angles (SAS), or when all three sides are proportional (SSS). A line parallel to one side divides the other two sides proportionally. In a right triangle, the altitude to the hypotenuse creates similar triangles and geometric means.

Standards: G-SRT.B.4, G-SRT.B.5 · **Level:** Grade 10 Geometry

How do you prove that triangles are similar?

Similar triangles have congruent corresponding angles and proportional corresponding sides. Three shortcuts prove similarity.

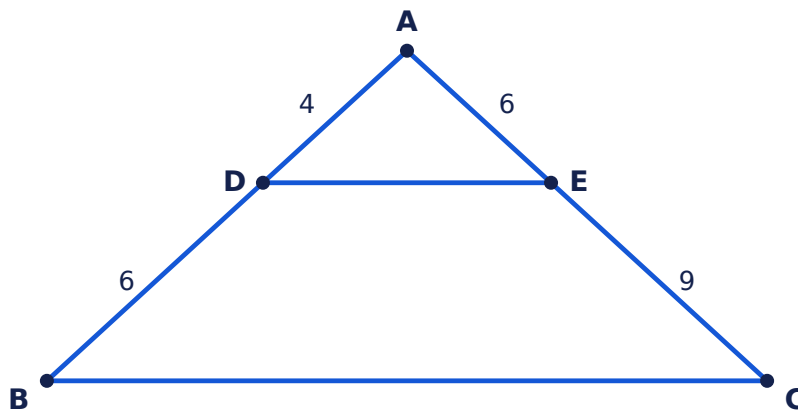
Criterion	What you need
AA	Two pairs of congruent angles
SAS ~	Two pairs of proportional sides and the congruent angles between them
SSS ~	Three pairs of proportional sides

With the scale factor k , every side of the second triangle is k times the matching side of the first. If one triangle has sides 7, 9, 12 and another has 10.5, 13.5, 18, the ratio is 1.5 each time, so the triangles are similar by SSS ~.

What does a line parallel to a side do?

Side-Splitter Theorem and converse

If a line parallel to one side of a triangle meets the other two sides, it divides them proportionally. Conversely, if a line divides two sides proportionally, it is parallel to the third side.



Side-Splitter Theorem: if $DE \parallel BC$, then $AD/DB = AE/EC$: $4/6 = 6/9$.

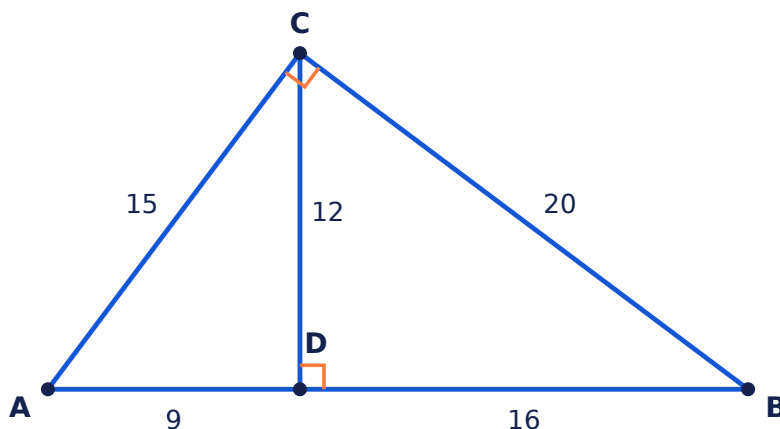
The small triangle ADE is similar to ABC by AA, because $\angle A$ is shared and the corresponding angles at D and B are congruent.

How is a right triangle split by its altitude?

The altitude to the hypotenuse creates two smaller right triangles. Each is similar to the original and to each other. This gives geometric mean relationships.

Right triangle relationships

- Altitude: $h^2 = p \cdot q$, so h is the geometric mean of the two pieces of the hypotenuse.
- Each leg: $a^2 = p \cdot c$ and $b^2 = q \cdot c$.

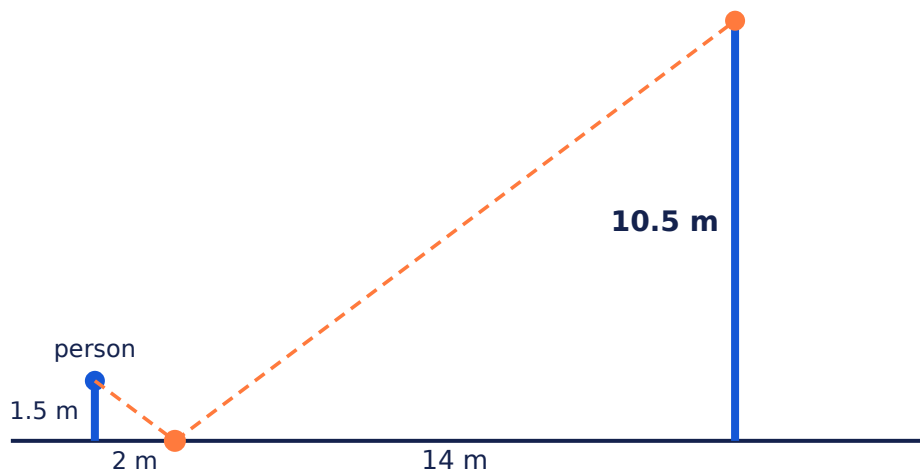


Altitude to the hypotenuse: $CD^2 = AD \cdot DB$, so $12^2 = 9 \cdot 16$.

Adding the two leg equations gives $a^2 + b^2 = c(p + q) = c^2$, which proves the Pythagorean Theorem using similar triangles. Check: $15^2 + 20^2 = 625 = 25^2$.

How do you measure something you cannot reach?

Build two similar triangles from distances you can measure, write a proportion, and solve. A mirror, a shadow or a pair of stakes can create the triangles.



Mirror method: similar right triangles give $h/14 = 1.5/2$, so $h = 10.5$ m.

Method

1. Find or prove two similar triangles and name them with matching vertices.
2. Write a proportion of corresponding sides.
3. Cross-multiply and solve.
4. Check that the answer has sensible size.

Common mistake

Match sides by their position in the triangle, not by how the figure looks. The proportion $AD/DB = AE/EC$ compares parts to parts, while $AD/AB = DE/BC$ compares part to whole.

Mathbot's tip

Write the similarity statement first, such as $\triangle ADE \sim \triangle ABC$. The order of the letters tells you which sides to put in each ratio.

Key points to remember

- AA, SAS and SSS similarity criteria prove triangles similar.
- A line parallel to a side divides the other two sides proportionally, and the converse is true.
- In a right triangle, the altitude to the hypotenuse satisfies $h^2 = p \cdot q$.
- Each leg is the geometric mean of the hypotenuse and the adjacent segment: $a^2 = p \cdot c$.
- Similar triangles let you measure heights and distances indirectly.

Similar Triangles, Proportions and Indirect Measurement: frequently asked questions

What is the difference between similar and congruent triangles?

Congruent triangles have the same size and shape. Similar triangles have the same shape and corresponding sides in a constant ratio, so they can be different sizes.

How do you find the geometric mean of two numbers?

Multiply the numbers and take the square root. The geometric mean of 4 and 25 is $\sqrt{100} = 10$.

Why does a line parallel to one side form a similar triangle?

Corresponding angles formed by the parallel line are congruent, and the angle at the top is shared. Two matching angle pairs make the triangles similar by AA.

How does similarity prove the Pythagorean Theorem?

The altitude to the hypotenuse gives $a^2 = p \cdot c$ and $b^2 = q \cdot c$. Adding them gives $a^2 + b^2 = c(p + q) = c^2$.

What is indirect measurement?

It is finding a length you cannot measure directly by using similar triangles. A shadow, a mirror or a pair of stakes creates two triangles with a known ratio.