

Triangle Theorems and Triangle Centers

Grade 10 – Geometry math · Lesson · Standards: G-CO.C.10, G-C.A.3

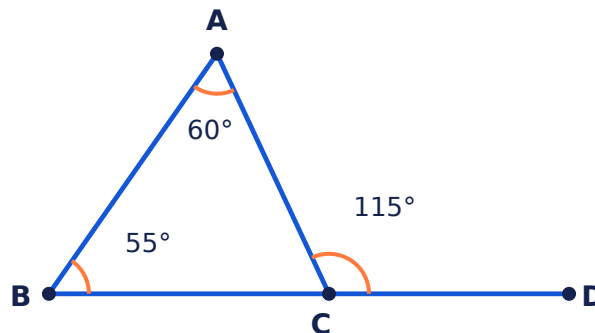
Quick answer: Triangle interior angles sum to 180° , and an exterior angle equals the sum of its two remote interior angles. A midsegment is parallel to the third side and half as long. The medians meet at the centroid, two thirds of the way from each vertex. Perpendicular bisectors, angle bisectors and altitudes meet at the circumcenter, incenter and orthocenter.

Standards: G-CO.C.10, G-C.A.3 · **Level:** Grade 10 Geometry

What do the angles of a triangle add up to?

Triangle theorems

- **Angle Sum:** the three interior angles add to 180° .
- **Exterior Angle Theorem:** an exterior angle equals the sum of the two remote interior angles.
- **Isosceles Triangle Theorem:** angles opposite congruent sides are congruent, and the converse is true.

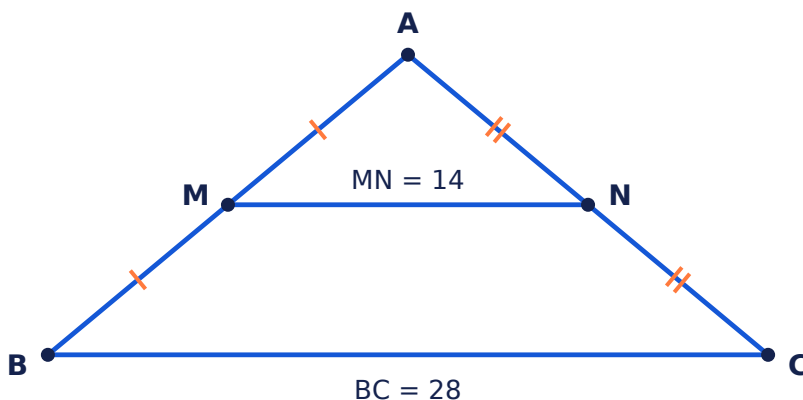


$$\text{Exterior angle } \angle ACD = \angle A + \angle B: 115^\circ = 60^\circ + 55^\circ.$$

The exterior angle forms a linear pair with the interior angle at the same vertex, which is why it equals the sum of the other two interior angles.

What does a midsegment tell us?

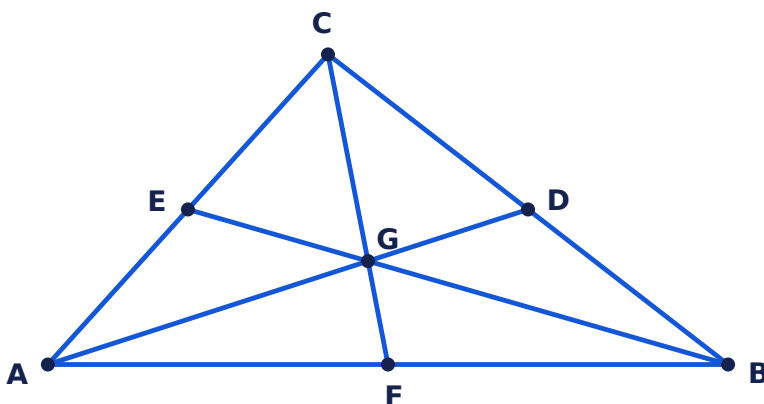
A **midsegment** joins the midpoints of two sides of a triangle. It is parallel to the third side and half as long. The three midsegments cut a triangle into four congruent triangles.



Midsegment: $MN \parallel BC$ and $MN = \frac{1}{2}BC$.

Where do special segments of a triangle meet?

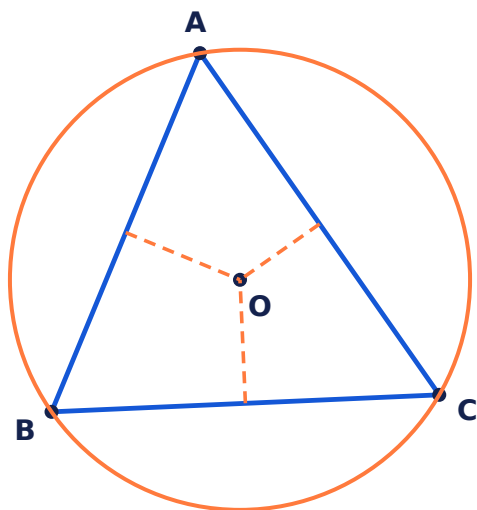
Point	Made by	Key fact
Centroid	Medians (vertex to midpoint of opposite side)	Splits each median 2 : 1 from the vertex
Circumcenter	Perpendicular bisectors of the sides	Equidistant from the three vertices
Incenter	Angle bisectors	Equidistant from the three sides
Orthocenter	Altitudes (vertex perpendicular to opposite side)	Inside, on or outside the triangle



Centroid G : $AG = 2GD$, so with $AD = 24$, $AG = 16$ and $GD = 8$.

What are inscribed and circumscribed circles?

The circumcenter is the center of the **circumscribed circle**, which passes through all three vertices. The incenter is the center of the **inscribed circle**, which touches all three sides. For a right triangle, the circumcenter is the midpoint of the hypotenuse.



Circumcenter O: the perpendicular bisectors meet at the center of the circumscribed circle, so $OA = OB = OC$.

Method for coordinates

1. Centroid: average the three x-coordinates and the three y-coordinates.
2. Circumcenter: find two perpendicular bisectors and intersect them.
3. Check that the distances to the vertices are equal.

Worked example

A median is 30 units long. The centroid is $\frac{2}{3}$ of the way from the vertex, so it lies $2 \cdot 30 \div 3 =$ **20 units** from the vertex and 10 from the midpoint.

Common mistake

The 2 : 1 ratio belongs to the medians only. Angle bisectors and altitudes do not split each other in that ratio.

Mathbot's tip

To remember the points: Centroid = Center of mass (medians), Circumcenter = Circle through vertices, Incenter = Inside circle.

Key points to remember

- Interior angles of a triangle add to 180° ; an exterior angle equals the sum of the remote interior angles.
- Base angles of an isosceles triangle are congruent, and congruent base angles mean congruent legs.
- A midsegment is parallel to the third side and half as long.

- The centroid divides each median 2 : 1; its coordinates are the averages of the vertices.
- The circumcenter is equidistant from vertices; the incenter is equidistant from sides.

Triangle Theorems and Triangle Centers: frequently asked questions

Why does the exterior angle equal the sum of the remote interior angles?

The exterior angle and its adjacent interior angle form a linear pair, so they add to 180° . The three interior angles also add to 180° , so the exterior angle must equal the other two interior angles together.

What is the difference between a median and an altitude?

A median runs from a vertex to the midpoint of the opposite side. An altitude runs from a vertex perpendicular to the line containing the opposite side. They coincide only in special triangles.

Where is the circumcenter of a right triangle?

It is the midpoint of the hypotenuse. The hypotenuse is a diameter of the circumscribed circle, so the midpoint is the same distance from all three vertices.

Can the orthocenter lie outside the triangle?

Yes. In an obtuse triangle the altitudes, extended as lines, meet outside the triangle. In a right triangle the orthocenter is the right-angle vertex.

How do you find the centroid from coordinates?

Average the x-coordinates of the vertices and average the y-coordinates. For vertices (0, 0), (6, 0) and (3, 9), the centroid is (3, 3).