

Triangle Theorems and Triangle Centers

Grade 10 – Geometry math · Test Answer Key · Standards: G-CO.C.10, G-C.A.3

Each solution shows the theorem, the work and the scoring split. Proof answers show one valid chain of reasons; any logically complete proof earns the same credit.

Standards: G-CO.C.10, G-C.A.3 · **Level:** Grade 10 Geometry

Question 1 Isosceles and exterior angle (2 points)

- $(180^\circ - 56^\circ) \div 2 = 62^\circ$.
- The exterior angle is supplementary to 62° : $180^\circ - 62^\circ = 118^\circ$. (Check: $56^\circ + 62^\circ = 118^\circ$.)

(a) 62° , (b) 118°

Scoring: 1 point per part.

Question 2 Exterior angle equation (3 points)

- $7x - 9 = (3x + 12) + (2x + 19) = 5x + 31$, so $2x = 40$ and $x = 20$.
- $7(20) - 9 = 131^\circ$.
- $3(20) + 12 = 72^\circ$ and $2(20) + 19 = 59^\circ$; $72 + 59 = 131$.

$x = 20$; 131° ; 72° and 59°

Scoring: 1 point per part.

Question 3 Midsegment equation (2 points)

$BC = 2MN$: $7x + 9 = 10x - 6$, so $15 = 3x$ and $x = 5$. Then $BC = 7(5) + 9 = 44$.

$x = 5$; $BC = 44$

Scoring: 1 point for x , 1 point for BC .

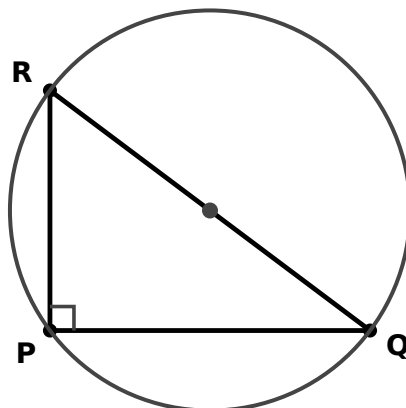
Question 4 Centroid (3 points)

- $(-3 + 9 + 0) \div 3 = 2$.
- $(5 + 1 - 6) \div 3 = 0$.
- The centroid is two thirds of the way from the vertex: $\frac{2}{3} \cdot 24 = 16$.

Centroid (2, 0); 16 units

Scoring: 1 point per part.

Question 5 Circumscribed circle (3 points)



Solved figure: circumcenter (4, 3), radius 5.

- The legs lie on the axes, so the right angle is at P .
- The circumcenter is the midpoint of the hypotenuse QR : $((8 + 0) \div 2, (0 + 6) \div 2) = (4, 3)$.
- $QR = \sqrt{(8^2 + 6^2)} = 10$, so the radius is 5.

Right angle at P; circumcenter (4, 3); radius 5

Scoring: 1 point per part.

Question 6 Prove a perpendicular (3 points)

Statement	Reason
1. $AB \cong AC$ and D is the midpoint of BC	Given
2. $BD \cong CD$	Definition of midpoint
3. $AD \cong AD$	Reflexive Property
4. $\triangle ABD \cong \triangle ACD$	SSS
5. $\angle ADB \cong \angle ADC$	CPCTC
6. $\angle ADB$ and $\angle ADC$ are a linear pair, so each is 90°	Congruent angles in a linear pair are right angles
7. $AD \perp BC$	Definition of perpendicular

Scoring: 1 point for the congruent triangles with reasons (steps 2 to 4), 1 point for step 5, 1 point for the right-angle conclusion.

Question 7 Inradius of a right triangle (2 points)

- a. $(5 + 12 + 13) \div 2 = 15$.
- b. $\text{Area} = \frac{1}{2} \cdot 5 \cdot 12 = 30$ and $30 = r \cdot 15$, so $r = 2$.

(a) 15, (b) $r = 2$

Scoring: 1 point per part.

Question 8 Garden planning (2 points)

Equal distance from the three corners (vertices) is the circumcenter, the intersection of the perpendicular bisectors. Equal distance from the three edges is the incenter, the intersection of the angle bisectors.

Spigot: circumcenter; bench: incenter

Scoring: 1 point for each correct center.