

Triangle Theorems and Triangle Centers

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-CO.C.10, G-C.A.3

Each answer names the theorem used, writes the equation and checks the result with a second relationship, such as an angle sum or a distance. Proofs list a reason beside every statement.

Standards: G-CO.C.10, G-C.A.3 · **Level:** Grade 10 Geometry

Problem 1 Third angle of a triangle

The interior angles add to 180° : $180^\circ - 47^\circ - 68^\circ = 65^\circ$.

65°

Problem 2 Exterior angle

By the Exterior Angle Theorem, $m\angle ACD = 52^\circ + 79^\circ = 131^\circ$.

Check: $\angle ACB = 180^\circ - 131^\circ = 49^\circ$, and $52 + 79 + 49 = 180$.

131°

Problem 3 Base angles

The two base angles are congruent and the three angles add to 180° , so each base angle is $(180^\circ - 38^\circ) \div 2 = 71^\circ$.

71° each

Problem 4 Converse of the isosceles theorem

Congruent angles at B and C mean the opposite sides AC and AB are congruent: $3x - 4 = x + 10$, so $2x = 14$ and $x = 7$.

$AB = 3(7) - 4 = 17$, and $AC = 7 + 10 = 17$.

$x = 7$; $AB = 17$

Problem 5 Midsegment length

A midsegment is half the length of the third side: $MN = \frac{1}{2}(26.4) = 13.2$.

$$MN = 13.2 \text{ cm}$$

Problem 6 Midsegment with algebra

$$BC = 2MN: 8x - 10 = 2(2x + 7) = 4x + 14, \text{ so } 4x = 24 \text{ and } x = 6.$$

$$MN = 2(6) + 7 = 19 \text{ and } BC = 8(6) - 10 = 38.$$

$$x = 6; MN = 19; BC = 38$$

Problem 7 Angle sum with algebra

$$(2x + 5) + (3x - 10) + (x + 35) = 180 \text{ gives } 6x + 30 = 180, \text{ so } x = 25.$$

The angles are $2(25) + 5 = 55^\circ$, $3(25) - 10 = 65^\circ$ and $25 + 35 = 60^\circ$. Check: $55 + 65 + 60 = 180$.

$$x = 25; 55^\circ, 65^\circ, 60^\circ$$

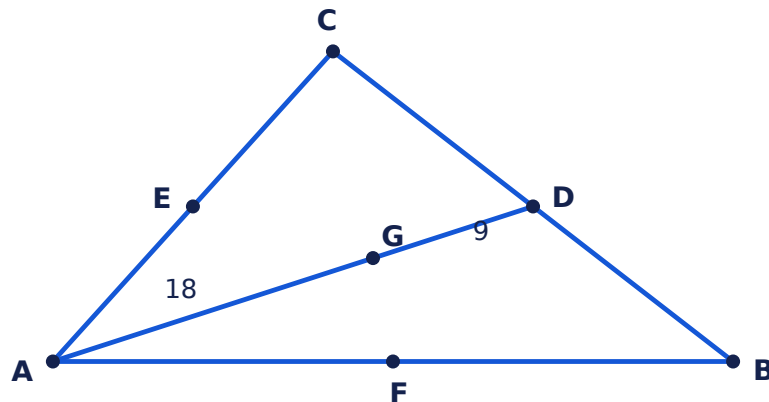
Problem 8 Exterior angle with algebra

The exterior angle equals the sum of the remote angles: $8x - 9 = (3x + 14) + (2x + 25) = 5x + 39$, so $3x = 48$ and $x = 16$.

The exterior angle is $8(16) - 9 = 119^\circ$, and the remote angles are 62° and 57° , which add to 119° .

$$x = 16; 119^\circ$$

Problem 9 Parts of a median



Solved figure: $AG = 18$ and $GD = 9$.

The centroid is two thirds of the way from the vertex: $AG = \frac{2}{3}(27) = 18$, and $GD = 27 - 18 = 9$

(half of AG).

AG = 18, GD = 9

Problem 10 Centroid ratio with algebra

$AG = 2GD$: $3x - 2 = 2(x + 4) = 2x + 8$, so $x = 10$.

$AG = 28$ and $GD = 14$, so the median is $28 + 14 = 42$.

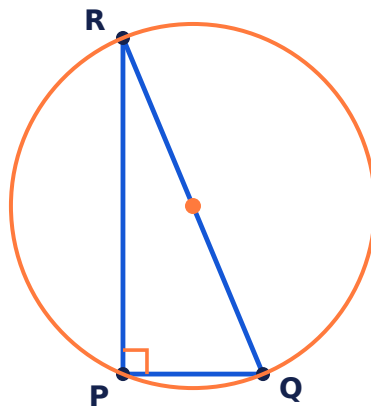
$x = 10$; AG = 28; median 42

Problem 11 Centroid coordinates

Average the coordinates: $x = (1 + 7 + 4) \div 3 = 4$ and $y = (2 - 1 + 8) \div 3 = 3$.

Centroid (4, 3)

Problem 12 Circumcenter of a right triangle



The circumcenter (5, 12) is the midpoint of the hypotenuse.

The right angle is at P , so the hypotenuse is QR . Its length is $\sqrt{(10^2 + 24^2)} = \sqrt{676} = 26$.

The circumcenter is the midpoint of QR : (5, 12). The radius is half the hypotenuse, 13. Check: distance to $P = \sqrt{(25 + 144)} = 13$.

Circumcenter (5, 12); radius 13

Problem 13 Inscribed circle radius

Area = $\frac{1}{2} \cdot 9 \cdot 12 = 54$. Semiperimeter = $(9 + 12 + 15) \div 2 = 18$. Then $54 = r \cdot 18$, so $r = 3$.

Check with the right-triangle formula: $(9 + 12 - 15) \div 2 = 3$.

$$r = 3$$

Problem 14 Name the point of concurrency

- a. Circumcenter, equidistant from the vertices.
- b. Incenter, equidistant from the sides.
- c. Centroid, the balance point.
- d. Orthocenter.

(a) circumcenter, (b) incenter, (c) centroid, (d) orthocenter

Problem 15 Fire station location

The station must be the circumcenter. The right angle is at A , so the circumcenter is the midpoint of hypotenuse BC : $(6, 8)$.

$BC = \sqrt{(12^2 + 16^2)} = 20$, so the distance is 10 miles. Check: distance to $A = \sqrt{(36 + 64)} = 10$.

Station at $(6, 8)$; 10 miles from each town

Problem 16 Balance point of a plate

A uniform triangular plate balances at its centroid. $x = (0 + 9 + 3) \div 3 = 4$ and $y = (0 + 0 + 12) \div 3 = 4$.

The support goes at $(4, 4)$

Problem 17 Perimeter from midsegments

Each midsegment is half of the side it is parallel to: 9, 13 and 11. The perimeter is $9 + 13 + 11 = 33$.

The perimeter of $\triangle ABC$ is $18 + 26 + 22 = 66$, so the small triangle has half the perimeter.

Sides 9, 13, 11; perimeter 33, half of 66

Problem 18 Prove the exterior angle theorem

Statement	Reason
1. $m\angle A + m\angle B + m\angle ACB = 180^\circ$	Triangle Angle Sum Theorem
2. $\angle ACB$ and $\angle ACD$ form a linear pair	Definition of linear pair
3. $m\angle ACB + m\angle ACD = 180^\circ$	Linear pairs are supplementary
4. $m\angle A + m\angle B + m\angle ACB = m\angle ACB + m\angle ACD$	Substitution

Statement	Reason
5. $m\angle ACD = m\angle A + m\angle B$	Subtraction Property of Equality

$$m\angle ACD = m\angle A + m\angle B$$

Problem 19 Prove the isosceles triangle theorem

Statement	Reason
1. $AB \cong AC$	Given
2. Draw the bisector AD of $\angle BAC$, so $\angle BAD \cong \angle CAD$	Definition of angle bisector
3. $AD \cong AD$	Reflexive Property
4. $\triangle ABD \cong \triangle ACD$	SAS
5. $\angle B \cong \angle C$	CPCTC

$$\angle B \cong \angle C$$

Problem 20 Midsegment on the coordinate plane

$$M = ((2 - 4) \div 2, (10 + 2) \div 2) = (-1, 6) \text{ and } N = ((2 + 10) \div 2, (10 + 0) \div 2) = (6, 5).$$

$$\text{Slope of } MN = (5 - 6) \div (6 - (-1)) = -1/7. \text{ Slope of } BC = (0 - 2) \div (10 - (-4)) = -2/14 = -1/7. \text{ Equal slopes, so } MN \parallel BC.$$

$$\text{Lengths: } MN = \sqrt{7^2 + 1^2} = \sqrt{50} \text{ and } BC = \sqrt{14^2 + 2^2} = \sqrt{200} = 2\sqrt{50}. \text{ So } MN = \frac{1}{2}BC.$$

$$\text{Both slopes are } -1/7; MN = \sqrt{50} = \frac{1}{2} \cdot BC$$