

# Triangle Theorems and Triangle Centers

Grade 10 – Geometry math · Worksheet · Standards: G-CO.C.10, G-C.A.3

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

These problems cover G-CO.C.10 and G-C.A.3: the triangle angle sum, exterior angles, isosceles triangles, midsegments, the centroid, circumcenter, incenter and orthocenter, and the inscribed and circumscribed circles of a triangle.

Early problems apply one theorem. Later problems combine algebra, coordinates and short proofs.

**Standards:** G-CO.C.10, G-C.A.3 · **Level:** Grade 10 Geometry

## Problem 1 Third angle of a triangle (easy)

Two angles of a triangle measure  $47^\circ$  and  $68^\circ$ . Find the third angle.

## Problem 2 Exterior angle (easy)

In  $\triangle ABC$ ,  $m\angle A = 52^\circ$  and  $m\angle B = 79^\circ$ . Side  $BC$  is extended to  $D$ . Find  $m\angle ACD$ .

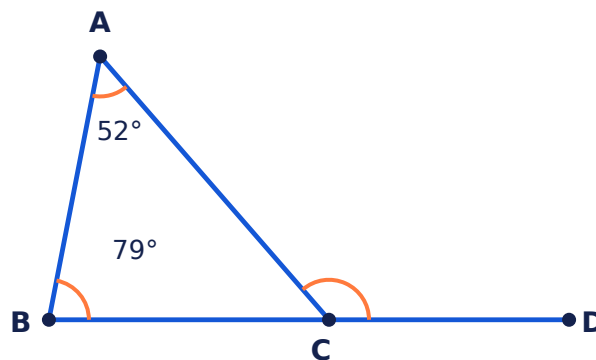


Figure for Problem 2.

## Problem 3 Base angles (easy)

The vertex angle of an isosceles triangle measures  $38^\circ$ . Find each base angle.

## Problem 4 Converse of the isosceles theorem (easy)

In  $\triangle ABC$ ,  $\angle B \cong \angle C$ ,  $AB = 3x - 4$  and  $AC = x + 10$ . Find  $x$  and  $AB$ .

**Problem 5 Midsegment length** (easy)

$M$  and  $N$  are the midpoints of  $AB$  and  $AC$ . If  $BC = 26.4$  centimeters, find  $MN$ .

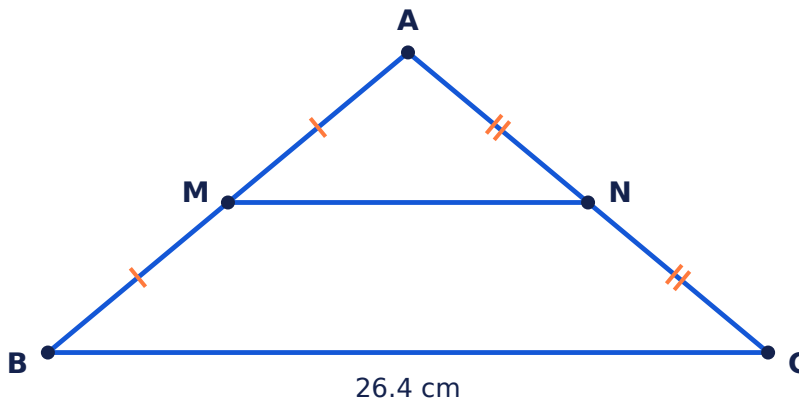


Figure for Problem 5.

**Problem 6 Midsegment with algebra** (medium)

In  $\triangle ABC$ ,  $MN$  joins the midpoints of  $AB$  and  $AC$ .  $MN = 2x + 7$  and  $BC = 8x - 10$ . Find  $x$ ,  $MN$  and  $BC$ .

**Problem 7 Angle sum with algebra** (medium)

The angles of a triangle measure  $(2x + 5)^\circ$ ,  $(3x - 10)^\circ$  and  $(x + 35)^\circ$ . Find  $x$  and all three angles.

**Problem 8 Exterior angle with algebra** (medium)

An exterior angle of a triangle measures  $(8x - 9)^\circ$ . The remote interior angles measure  $(3x + 14)^\circ$  and  $(2x + 25)^\circ$ . Find  $x$  and the exterior angle.

**Problem 9 Parts of a median** (medium)

The centroid  $G$  lies on median  $AD$ , and  $AD = 27$ . Find  $AG$  and  $GD$ .

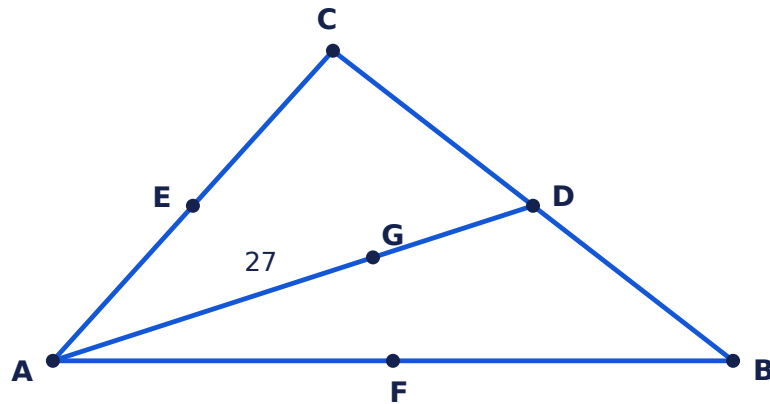


Figure for Problem 9.

### Problem 10 Centroid ratio with algebra (medium)

The medians of a triangle meet at G. On one median,  $AG = 3x - 2$  and  $GD = x + 4$ . Find  $x$ , AG and the length of the whole median.

### Problem 11 Centroid coordinates (medium)

Triangle ABC has  $A(1, 2)$ ,  $B(7, -1)$  and  $C(4, 8)$ . Find the centroid.

### Problem 12 Circumcenter of a right triangle (medium)

Triangle PQR has  $P(0, 0)$ ,  $Q(10, 0)$  and  $R(0, 24)$ . Find the circumcenter and the radius of the circumscribed circle.

### Problem 13 Inscribed circle radius (medium)

A right triangle has legs of 9 and 12 and a hypotenuse of 15. Find the radius of its inscribed circle using  $\text{area} = \text{radius} \times \text{semiperimeter}$ .

### Problem 14 Name the point of concurrency (medium)

Name the point of concurrency described by each statement.

- The point where the three perpendicular bisectors meet.
- The point where the three angle bisectors meet.
- The point where the three medians meet.
- The point where the three altitudes meet.

**Problem 15 Fire station location** (hard)

Three towns sit at  $A(0, 0)$ ,  $B(12, 0)$  and  $C(0, 16)$ , with each unit equal to one mile. A county plans a fire station the same distance from all three towns. Find its location and that distance.

**Problem 16 Balance point of a plate** (hard)

A robotics club cuts a triangular aluminum plate with vertices  $(0, 0)$ ,  $(9, 0)$  and  $(3, 12)$ , measured in centimeters. Where should a single support rod go to balance the plate?

**Problem 17 Perimeter from midsegments** (hard)

The sides of  $\triangle ABC$  are  $AB = 18$ ,  $BC = 26$  and  $CA = 22$ . The midpoints of the three sides form a smaller triangle. Find its three side lengths and its perimeter, and compare with the perimeter of  $\triangle ABC$ .

**Problem 18 Prove the exterior angle theorem** (hard)

In  $\triangle ABC$ , side  $BC$  is extended to  $D$ . Prove that  $m\angle ACD = m\angle A + m\angle B$ . Write a two-column proof.

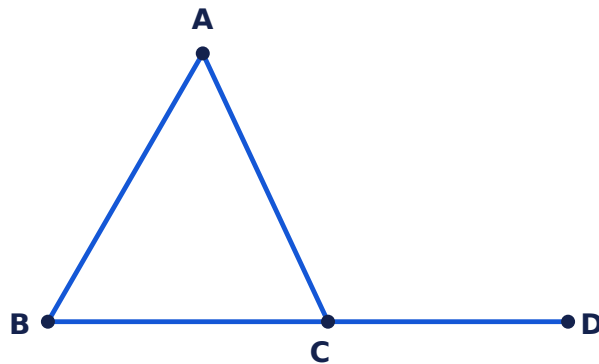


Figure for Problem 18.

**Problem 19 Prove the isosceles triangle theorem** (hard)

Given:  $AB \cong AC$  in  $\triangle ABC$ . Prove:  $\angle B \cong \angle C$ . Draw the bisector of  $\angle A$  meeting  $BC$  at  $D$ , then write a two-column proof.

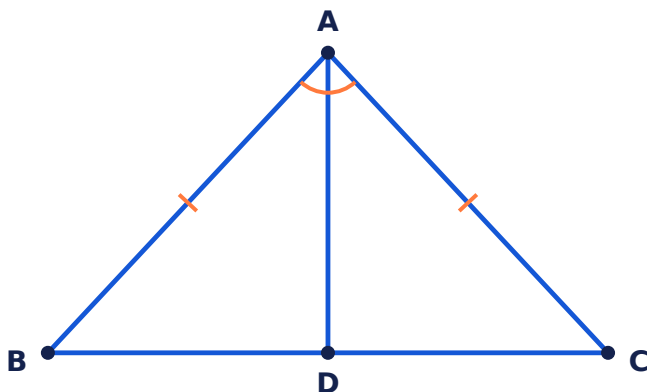


Figure for Problem 19.

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**Problem 20 Midsegment on the coordinate plane** (hard)

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Triangle  $ABC$  has  $A(2, 10)$ ,  $B(-4, 2)$  and  $C(10, 0)$ . Let  $M$  and  $N$  be the midpoints of  $AB$  and  $AC$ . Show that  $MN$  is parallel to  $BC$  and half as long.