

Reasoning, Logic and Two-Column Proofs

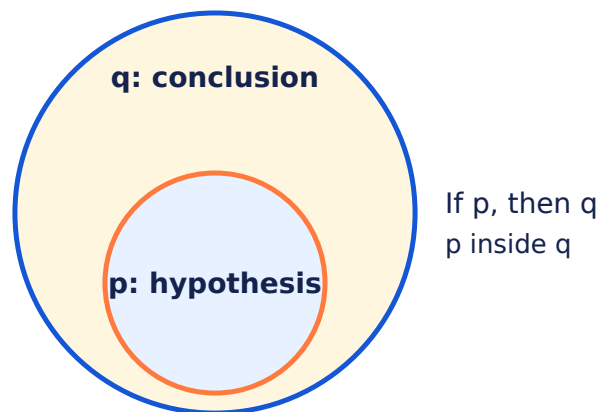
Grade 10 – Geometry math · Lesson · Standards: G-CO.C.9

Quick answer: A conditional statement says if p , then q . Its converse, inverse and contrapositive rearrange p and q , and only the contrapositive always matches the original truth value. Deductive reasoning builds proofs from givens, definitions and properties of equality. A two-column proof lists each statement with a reason, ending at the statement to prove.

Standards: G-CO.C.9 · **Level:** Grade 10 Geometry

What does an if-then statement say?

An **if-then statement**, also called a conditional, links a hypothesis to a conclusion: "if p , then q ." The **hypothesis** p follows "if" and the **conclusion** q follows "then." A conditional is false only when a **counterexample** exists: a case where p is true and q is false.

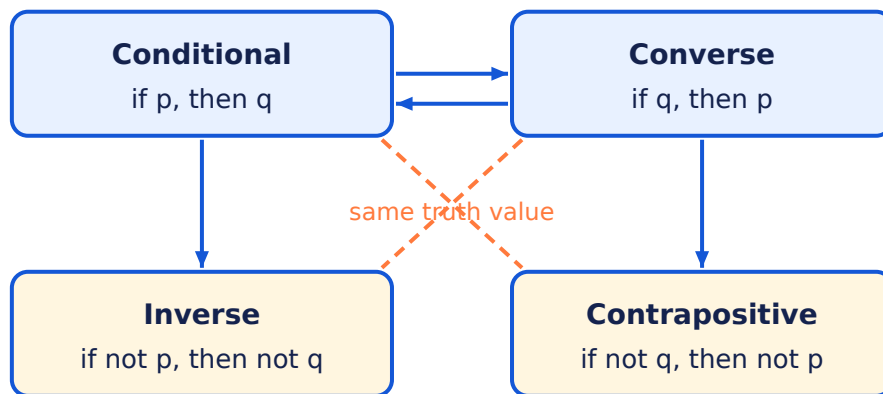


A conditional says the set described by p lies completely inside the set described by q .

How are the converse, inverse and contrapositive formed?

Related statements

- **Converse:** swap the parts, so if q , then p .
- **Inverse:** negate both parts, so if not p , then not q .
- **Contrapositive:** swap and negate, so if not q , then not p .



Dashed lines join statements that are always true together or false together.

The contrapositive of a true statement is always true. The converse is true only some of the time. If both directions hold, merge them into a **biconditional**: " p if and only if q ."

How does inductive reasoning differ from deductive reasoning?

Inductive reasoning spots a pattern in examples and makes a conjecture; one counterexample can still break it. **Deductive reasoning** applies accepted facts, step by step, to reach a conclusion that must be true.

Which properties of equality can a proof cite?

Property	Meaning
Reflexive	$a = a$
Symmetric	If $a = b$, then $b = a$
Transitive	If $a = b$ and $b = c$, then $a = c$
Addition / Subtraction	Add or subtract the same number on both sides
Multiplication / Division	Multiply or divide both sides by the same nonzero number
Substitution	Replace a quantity by an equal one
Distributive	$a(b + c) = ab + ac$

How is a two-column proof built?

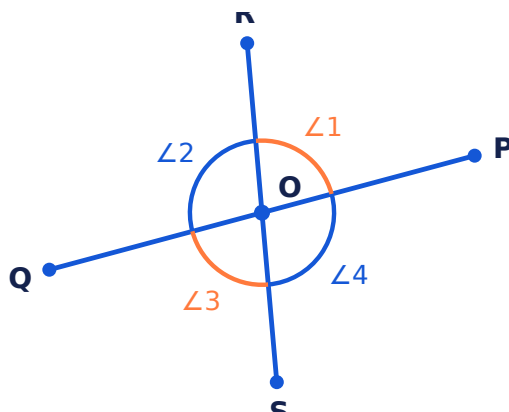
Method

1. Mark the Given facts and the Prove statement on a figure.
2. Plan a chain of statements from the Given to the Prove.
3. Place each statement on the left side of the table.
4. Write a reason in the right column: Given, a definition, a postulate, a theorem or a property of equality.
5. End with the Prove statement.

Vertical Angles Theorem

Given: $\angle 1$ and $\angle 3$ are vertical angles. Prove: $\angle 1 \cong \angle 3$.

Statement	Reason
1. $\angle 1$ and $\angle 2$ form a linear pair	Given (from the figure)
2. $\angle 2$ and $\angle 3$ form a linear pair	Given (from the figure)
3. $m\angle 1 + m\angle 2 = 180^\circ$	Linear pairs are supplementary
4. $m\angle 2 + m\angle 3 = 180^\circ$	Linear pairs are supplementary
5. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	Substitution
6. $m\angle 1 = m\angle 3$	Subtraction Property of Equality



Angles $\angle 1$ and $\angle 3$ face each other across the crossing point.

Common mistake

Never use a picture as a reason. Marks on a figure can be read as Given, but a measurement or a congruence that merely looks true needs its own justification.

Mathbot's tip

Work backward from the Prove line. Ask which statement would make the last step true, then repeat until you reach the Given.

Key points to remember

- A conditional is false when one counterexample makes p true and q false.
- A conditional and its contrapositive are always true or false together.
- Inductive reasoning finds patterns; deductive reasoning proves them.
- Every step of a proof needs a reason: given, definition, postulate, theorem or property.
- Vertical angles are congruent, proved with two linear pairs.

Reasoning, Logic and Two-Column Proofs: frequently asked questions

How do you find a counterexample to a conditional statement?

Look for one case where the hypothesis is true and the conclusion is false. A single case is enough to prove the whole statement false.

Is the converse of a true statement always true?

No. "If a shape is a square, then it has four sides" is true, but its converse is false because a rectangle also has four sides.

What is the difference between a theorem and a postulate?

A postulate is accepted without proof. A theorem is a statement that has been proved from postulates, definitions and earlier theorems.

Why can a proof not rely on a picture?

A figure may be drawn out of scale or show a special case. Only facts that are given or proved can be used as reasons.

What is a paragraph proof?

A paragraph proof presents the same logical chain as a two-column proof, written in sentences. Each claim still needs a reason.