

Reasoning, Logic and Two-Column Proofs

Grade 10 – Geometry math · Worksheet Answer Key · Standards: G-CO.C.9

Each solution shows the statement, the reason behind it and the final result. In proofs, compare the order of your statements with the answer key and check that every line cites a justified reason.

Standards: G-CO.C.9 · **Level:** Grade 10 Geometry

Problem 1 Hypothesis and conclusion

The hypothesis follows "if" and the conclusion follows "then."

Hypothesis: a whole number ends in 0. Conclusion: it is divisible by 10.

Problem 2 Writing a converse

Swap the parts: "If a quadrilateral has four right angles, then it is a square." A rectangle that is not a square has four right angles, so the converse fails.

Converse: if a quadrilateral has four right angles, then it is a square. It is false.

Problem 3 Inverse and contrapositive

Inverse: negate both parts. Contrapositive: swap and negate.

The inverse is false because a sparrow is not a robin but is a bird. The contrapositive is true because anything that is not a bird cannot be a robin.

Inverse: if an animal is not a robin, then it is not a bird (false). Contrapositive: if an animal is not a bird, then it is not a robin (true).

Problem 4 Find a counterexample

The number 2 is prime (its only factors are 1 and 2), but 2 is even. The hypothesis is true and the conclusion is false.

Counterexample: 2

Problem 5 Spot the pattern

- a. The differences are 5, 7, 9, which increase by 2 each time.
- b. The next differences should be 11 and 13, so the next terms are $24 + 11 = 35$ and $35 + 13 = 48$. (Each term is a perfect square minus 1: $7^2 - 1 = 48$.)

Next two terms: 35 and 48

Problem 6 Inductive or deductive

- a. She generalizes from examples: inductive.
- b. A general fact is applied to one case: deductive.
- c. Generalizing from three triangles: inductive.
- d. A proven fact forces the value $180 - 70 - 60 = 50$: deductive.

(a) inductive, (b) deductive, (c) inductive, (d) deductive

Problem 7 Name the property

- a. Symmetric Property.
- b. Reflexive Property.
- c. Transitive Property (or Substitution).
- d. Distributive Property.
- e. Division Property of Equality.

(a) symmetric, (b) reflexive, (c) transitive, (d) distributive, (e) division

Problem 8 Reasons for solving steps

Statement	Reason
$4x - 12 + 5 = 2x + 9$	Distributive Property
$4x - 7 = 2x + 9$	Simplify (combine like terms)
$2x - 7 = 9$	Subtraction Property (subtract $2x$)
$2x = 16$	Addition Property (add 7)
$x = 8$	Division Property (divide by 2)

Check: $4(5) + 5 = 25$ and $2(8) + 9 = 25$.

$x = 8$

Problem 9 Biconditional statements

- a. Converse: if $x = 7$, then $3x + 1 = 22$. Both are true, so: $3x + 1 = 22$ if and only if $x = 7$.
- b. Converse: if a whole number is even, then it is divisible by 4. Counterexample: 6. The

converse is false, so no biconditional exists.

(a) biconditional is true; (b) no biconditional, since 6 is even but not divisible by 4

Problem 10 Given and Prove

Given: $\angle 1$ and $\angle 3$ are supplementary, and $\angle 2$ and $\angle 3$ are supplementary. Prove: $m\angle 1 = m\angle 2$.

Check: $m\angle 1 = 180^\circ - 64^\circ = 116^\circ$ and $m\angle 2 = 180^\circ - 64^\circ = 116^\circ$, so they are equal.

Given: $\angle 1, \angle 3$ supplementary; $\angle 2, \angle 3$ supplementary. Prove: $m\angle 1 = m\angle 2$ (both 116° when $\angle 3 = 64^\circ$).

Problem 11 Complete a segment proof

Statement	Reason
1. M is the midpoint of AB	Given
2. $AM = MB$	Definition of midpoint
3. $5x - 3 = 3x + 9$	Substitution
4. $2x - 3 = 9$	Subtraction Property
5. $2x = 12$	Addition Property
6. $x = 6$	Division Property

Check: $AM = 5(6) - 3 = 27$ and $MB = 3(6) + 9 = 27$.

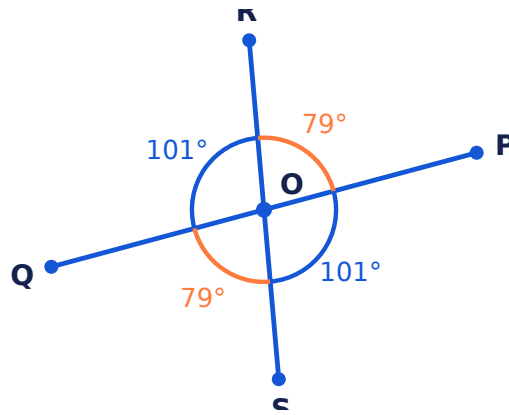
$x = 6$

Problem 12 Complements of one angle

Statement	Reason
1. $\angle AOB$ and $\angle BOC$ are complementary	Given
2. $\angle BOC$ and $\angle COD$ are complementary	Given
3. $m\angle AOB + m\angle BOC = 90^\circ$	Definition of complementary
4. $m\angle BOC + m\angle COD = 90^\circ$	Definition of complementary
5. $m\angle AOB + m\angle BOC = m\angle BOC + m\angle COD$	Substitution (both equal 90°)
6. $m\angle AOB = m\angle COD$	Subtraction Property

$m\angle AOB = m\angle COD$

Problem 13 Use the vertical angles theorem



Solved figure for Problem 13.

By the Vertical Angles Theorem, $8x - 17 = 5x + 19$, so $3x = 36$ and $x = 12$.

Each vertical angle is $8(12) - 17 = 79^\circ$. A neighbor forms a linear pair with 79° , so it measures $180^\circ - 79^\circ = 101^\circ$.

$x = 12$; 79° and 79° ; neighbors 101°

Problem 14 Paragraph proof of perpendicular lines

Because $\angle POR$ and $\angle ROQ$ sit on a straight line, they are supplementary, so $m\angle POR + m\angle ROQ = 180^\circ$. Since the angles are congruent, their measures are equal, so substitution gives $2m\angle POR = 180^\circ$. Dividing by 2 gives $m\angle POR = 90^\circ$. The lines meet at a right angle, so by definition they are perpendicular.

The lines are perpendicular because both angles measure 90° .

Problem 15 Camp rules and logic

- The hypothesis is true, so the range is closed.
- This is the contrapositive: not closed means not raining. It is not raining.
- No. The range may close for wind or a staff meeting, so the conclusion cannot be drawn.
- Converse: if the archery range is closed, then it is raining. The rule does not guarantee it.

(a) closed, (b) not raining, (c) no conclusion, (d) converse not guaranteed

Problem 16 Disproving three statements

- Angles of 100° and 80° drawn in different places are supplementary but not adjacent, so they are not a linear pair.
- $x = -6$, since $(-6)^2 = 36$ but $-6 \neq 6$.
- 20 is divisible by 5 and ends in 0.

(a) 100° and 80° apart, (b) -6 , (c) 20

Problem 17 Odd numbers and a false conjecture

Sums: 1, $1 + 3 = 4$, $1 + 3 + 5 = 9$, 16, 25. Conjecture: the sum of the first n odd numbers is n^2 . For $n = 12$ the prediction is 144.

For the second conjecture, $n = 41$ gives $41^2 - 41 + 41 = 41^2 = 1,681 = 41 \cdot 41$, which is not prime. So the conjecture is false even though it works for many smaller values.

Sum = n^2 (144 for $n = 12$); counterexample $n = 41$ gives $1,681 = 41 \cdot 41$

Problem 18 Linear pair and bisector steps

Statement	Reason
1. $\angle AOB$ and $\angle BOC$ form a linear pair	Given
2. $(7x + 11) + (3x + 19) = 180$	Linear pairs are supplementary
3. $10x + 30 = 180$, so $x = 15$	Simplify and Division Property
4. $m\angle AOB = 116^\circ$, $m\angle BOC = 64^\circ$	Substitution
5. $m\angle BOD = 64^\circ \div 2 = 32^\circ$	Definition of angle bisector

Check: $116 + 64 = 180$.

$x = 15$; 116° and 64° ; $m\angle BOD = 32^\circ$

Problem 19 Equal segments with a shared piece

Statement	Reason
1. $AB = CD$	Given
2. $BC = BC$	Reflexive Property
3. $AB + BC = CD + BC$	Addition Property
4. $AB + BC = AC$	Segment Addition Postulate
5. $BC + CD = BD$	Segment Addition Postulate
6. $AC = BD$	Substitution

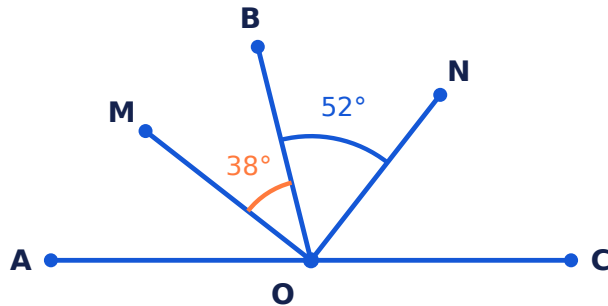
$AC = BD$

Problem 20 Bisectors of a linear pair

Statement	Reason
1. $\angle AOB$ and $\angle BOC$ form a linear pair	Given
2. $m\angle AOB + m\angle BOC = 180^\circ$	Linear pairs are supplementary
3. $m\angle MOB = \frac{1}{2}m\angle AOB$ and $m\angle BON = \frac{1}{2}m\angle BOC$	Definition of angle bisector
4. $m\angle MON = m\angle MOB + m\angle BON$	Angle Addition Postulate

Statement	Reason
5. $m\angle MON = \frac{1}{2}(m\angle AOB + m\angle BOC)$	Substitution and Distributive Property
6. $m\angle MON = \frac{1}{2}(180^\circ) = 90^\circ$	Substitution

Verification: if $m\angle AOB = 76^\circ$, then $m\angle BOC = 104^\circ$, so $m\angle MOB = 38^\circ$ and $m\angle BON = 52^\circ$, and $38^\circ + 52^\circ = 90^\circ$.



Solved figure: $\angle MOB = 38^\circ$ and $\angle BON = 52^\circ$.

$\angle MON = 90^\circ$