

Reasoning, Logic and Two-Column Proofs

Grade 10 – Geometry math · Worksheet · Standards: G-CO.C.9

Name: _____ Date: _____ Class: _____

These problems build the reasoning needed for G-CO.C.9: forming converse, inverse and contrapositive statements, telling inductive from deductive arguments, citing properties of equality and writing proofs.

The first problems are one-step logic tasks. Later problems ask for complete two-column and paragraph proofs, including the Vertical Angles Theorem.

Standards: G-CO.C.9 · **Level:** Grade 10 Geometry

Problem 1 Hypothesis and conclusion (easy)

Separate the following statement into its hypothesis and its conclusion: "If a whole number ends in 0, then it is divisible by 10."

Problem 2 Writing a converse (easy)

Form the converse of "If a quadrilateral is a square, then it has four right angles." Then judge whether that converse holds.

Problem 3 Inverse and contrapositive (easy)

Form the inverse and the contrapositive of "If an animal is a robin, then it is a bird." State the truth value of each.

Problem 4 Find a counterexample (easy)

Disprove "If a number is prime, then it is odd" with a single counterexample.

Problem 5 Spot the pattern (easy)

Look at the sequence below.



Sequence for Problem 5.

- Find the differences between consecutive terms.
- Make a conjecture for the next two terms.

Problem 6 Inductive or deductive (medium)

Label each argument inductive or deductive.

- Priya has met five golden retrievers, and all were friendly, so she concludes every golden retriever is friendly.
- All squares are rectangles. Figure $ABCD$ is a square. So $ABCD$ is a rectangle.
- A student measures the base angles of three isosceles triangles, finds them equal each time, and concludes base angles are always equal.
- The angles of a triangle add to 180° . Triangle PQR has $m\angle P = 70^\circ$ and $m\angle Q = 60^\circ$. So $m\angle R = 50^\circ$.

Problem 7 Name the property (medium)

Match each statement with the property of equality behind it.

- If $AB = 9$, then $9 = AB$.
- $m\angle P = m\angle P$.
- If $m\angle 1 = m\angle 2$ and $m\angle 2 = 47^\circ$, then $m\angle 1 = 47^\circ$.
- $3(2x - 5) = 6x - 15$.
- If $5x = 40$, then $x = 8$.

Problem 8 Reasons for solving steps (medium)

Supply a reason for every line when solving $4(x - 3) + 5 = 2x + 9$.

Statement	Reason
$4x - 12 + 5 = 2x + 9$	
$4x - 7 = 2x + 9$	
$2x - 7 = 9$	
$2x = 16$	
$x = 8$	

Problem 9 Biconditional statements (medium)

For each conditional, write the converse. If both are true, write a biconditional; if not, say why none exists.

- a. If $3x + 1 = 22$, then $x = 7$.
- b. If a whole number is divisible by 4, then it is even.

Problem 10 Given and Prove (medium)

Rewrite the theorem "Angles supplementary to the same angle have equal measures" with a Given and a Prove statement. Then check it with numbers when the shared angle $\angle 3$ measures 64° .

Problem 11 Complete a segment proof (medium)

Fill in the missing reasons. Given: point M bisects segment AB , $AM = 5x - 3$ and $MB = 3x + 9$. Prove that $x = 6$.

Statement	Reason
1. M is the midpoint of AB	
2. $AM = MB$	
3. $5x - 3 = 3x + 9$	
4. $2x - 3 = 9$	
5. $2x = 12$	
6. $x = 6$	

Problem 12 Complements of one angle (medium)

Given: $\angle AOB$ and $\angle BOC$ are complementary, and $\angle BOC$ and $\angle COD$ are complementary. Prove: $m\angle AOB = m\angle COD$. Write a two-column proof.

Problem 13 Use the vertical angles theorem (medium)

Two lines cross. A pair of vertical angles measures $(8x - 17)^\circ$ and $(5x + 19)^\circ$. Find x , the measure of each vertical angle, and the measure of a neighboring angle.

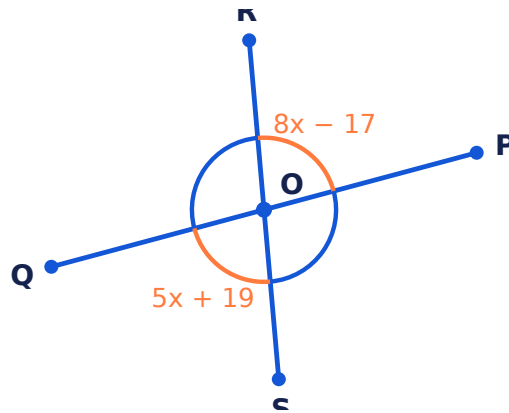


Figure for Problem 13.

Problem 14 Paragraph proof of perpendicular lines (medium)

Lines m and n cross, and two neighboring angles $\angle POR$ and $\angle ROQ$ share a side while their outer sides form a line. Given: $\angle POR \cong \angle ROQ$. Prove: $m \perp n$. Present the argument as a paragraph.

Problem 15 Camp rules and logic (hard)

A summer camp posts the rule: "If it is raining, then the archery range is closed." Assume the rule is true.

- It is raining on Tuesday. What follows?
- The range is open on Wednesday. What follows?
- The range is closed on Thursday. Can anyone conclude it is raining?
- Write the converse of the rule and say whether the rule guarantees it.

Problem 16 Disproving three statements (hard)

Give a counterexample for each statement.

- Whenever two angles add to 180° , they are neighbors on a line.
- If $x^2 = 36$, then $x = 6$.
- If a whole number is divisible by 5, then it ends in 5.

Problem 17 Odd numbers and a false conjecture (hard)

Add the first n odd numbers for $n = 1$ through 5 and state a conjecture. Use it to predict the sum of the first 12 odd numbers. Then test the conjecture " $n^2 - n + 41$ is prime for every whole number n " with $n = 41$.

Problem 18 Linear pair and bisector steps (hard)

$\angle AOB$ and $\angle BOC$ form a linear pair, with $m\angle AOB = (7x + 11)^\circ$ and $m\angle BOC = (3x + 19)^\circ$. Ray

OD bisects $\angle BOC$. Find x , both angles, and $m\angle BOD$.

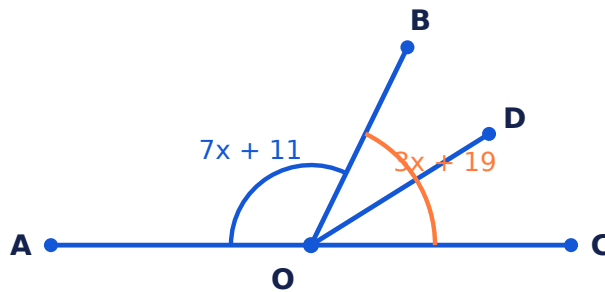


Figure for Problem 18.

Problem 19 Equal segments with a shared piece (hard)

Points A , B , C and D lie on a line in that order. Given: $AB = CD$. Prove: $AC = BD$. Write a two-column proof.



Figure for Problem 19: AB and CD are marked equal.

Problem 20 Bisectors of a linear pair (hard)

$\angle AOB$ and $\angle BOC$ form a linear pair. Ray OM bisects $\angle AOB$ and ray ON bisects $\angle BOC$. Prove that $\angle MON$ is a right angle. Then verify with $m\angle AOB = 76^\circ$.

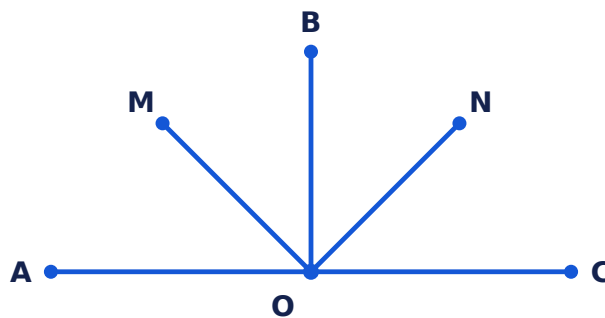


Figure for Problem 20.