

Volumes of Cylinders, Cones, Pyramids and Spheres

Grade 10 – Geometry math · Lesson · Standards: G-GMD.A.1, G-GMD.A.3

Quick answer: Volume formulas follow from slicing arguments. A cylinder holds $\pi r^2 h$, and by Cavalieri's principle a cone or pyramid holds one third of the matching cylinder or prism. A sphere holds four thirds πr^3 . These formulas let you find the capacity of tanks, silos and cones.

Standards: G-GMD.A.1, G-GMD.A.3 · **Level:** Grade 10 Geometry

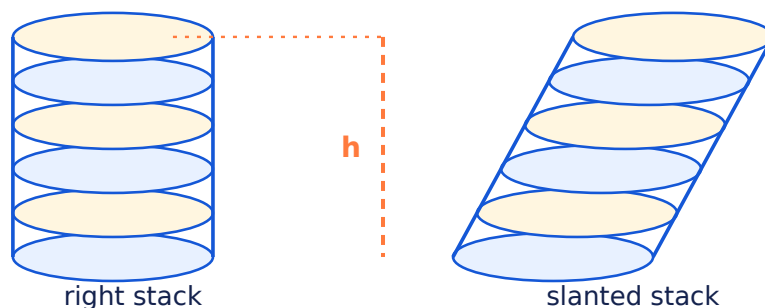
How can you see why a circle has area πr^2 ?

Cut a circle into thin wedges and lay them side by side, alternating up and down. The shape looks like a parallelogram whose base is half the circumference, πr , and whose height is the radius r . Its area is $\pi r \cdot r = \pi r^2$. The thinner the wedges, the closer the match.

What is Cavalieri's principle?

Cavalieri's principle

If two solids have the same height and every horizontal slice at the same level has the same area, then the solids have the same volume.

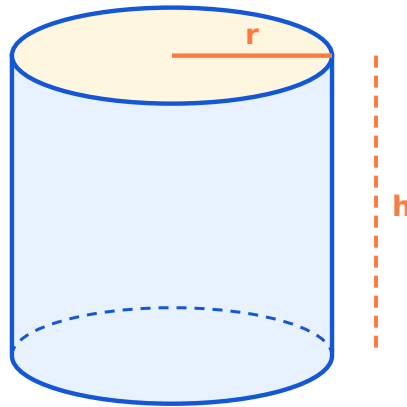


A straight stack and a slanted stack of the same coins have equal volumes.

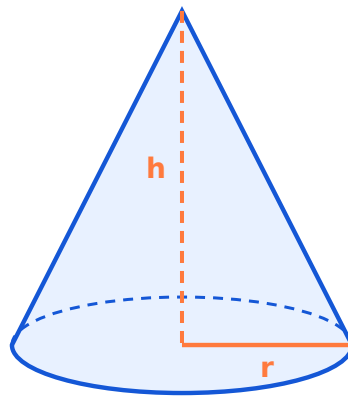
That is why an oblique cylinder has the same volume as a right cylinder with the same base and height, and why any pyramid or cone has one third of the volume of the prism or cylinder with the same base and height.

What are the volume formulas?

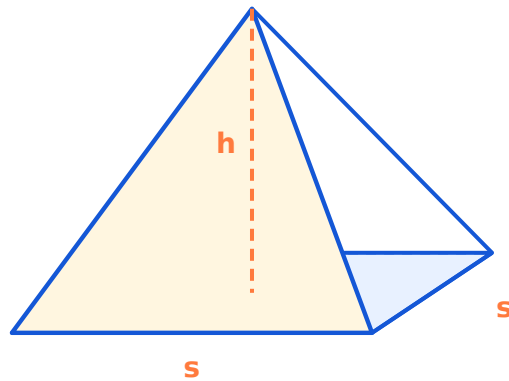
Solid	Volume
Cylinder	$\pi r^2 h$
Cone	$\frac{1}{3}\pi r^2 h$
Pyramid	$\frac{1}{3} (\text{area of base}) \cdot h$
Sphere	$\frac{4}{3}\pi r^3$



Cylinder: $V = \pi r^2 h$.



Cone: the height h is perpendicular to the base, not the slant.



Pyramid: $V = \frac{1}{3} \cdot s^2 \cdot h$.

Method

1. Sketch the solid and mark the radius or base sides and the perpendicular height.
2. Choose the formula that matches the solid.
3. Substitute, keeping π exact until the last step.
4. Attach cubic units, and convert if the question asks for liters or gallons.

Example

A cone has radius 9 cm and height 14 cm. Its volume is $\frac{1}{3}\pi(9)^2(14) = \frac{1}{3}\pi \cdot 1,134 = 378\pi \approx 1,187.5 \text{ cm}^3$.

Common mistake

Using the slant height instead of the perpendicular height. Volume formulas always need the height measured straight up from the base.

Mathbot's tip A cone or pyramid is one third of its matching cylinder or prism. If your answer is bigger than that matching solid, the one third is missing.

Key points to remember

- Cylinder: $V = \pi r^2 h$; cone: $V = \frac{1}{3}\pi r^2 h$; pyramid: $V = \frac{1}{3}Bh$.
- Sphere: $V = \frac{4}{3}\pi r^3$; a hemisphere is half of that.
- Cavalieri's principle: equal heights and equal cross-sectional areas mean equal volumes.
- Use the perpendicular height, never the slant height, in volume formulas.

Volumes of Cylinders, Cones, Pyramids and Spheres: frequently

asked questions

What is Cavalieri's principle?

It says that two solids of equal height have equal volume if every pair of slices at the same height has equal area. It explains why slanted and upright prisms or cylinders with the same base and height hold the same amount.

Why does a cone hold one third of a cylinder?

Three congruent pyramids can fill a cube, and slicing arguments extend this to any pyramid and cone. Each has one third the volume of the prism or cylinder with the same base and height.

How is the volume of a sphere found?

Compare a hemisphere with a cylinder that has a cone removed from it. Each slice has the same area at every height, so by Cavalieri's principle the hemisphere has volume $\frac{2}{3} \pi r^3$ and the sphere has $\frac{4}{3} \pi r^3$.

What is the difference between height and slant height?

The height is measured perpendicular to the base. The slant height runs along the lateral surface. Volume uses the height, and a right triangle connects the two.

How do I convert cubic centimeters to liters?

One liter is 1,000 cubic centimeters. Divide the volume in cubic centimeters by 1,000 to get liters.