

Compound Events, Sample Spaces and Simulations

Grade 7 math · Lesson · Standards: 7.SP.C.8

Quick answer: The sample space of a compound event is the complete list of its possible outcomes, found with a list, table or tree diagram. For equally likely outcomes, probability is matching outcomes divided by total outcomes. A simulation estimates probability by repeating a random device many times.

Standards: 7.SP.C.8 · **Level:** Grade 7

What is a sample space?

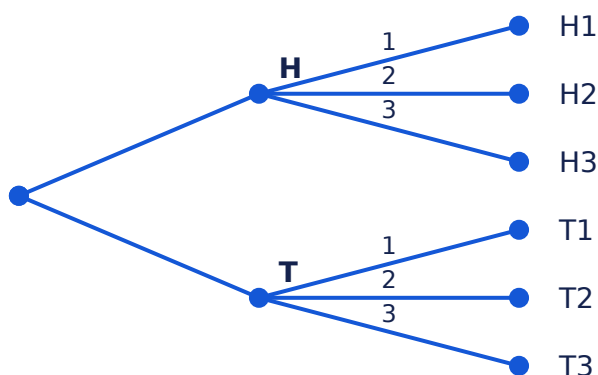
A compound event combines two or more simple events, such as flipping a coin and then spinning a spinner. The **sample space** is the complete list of possible outcomes. Making the list in an organized way, with a table or a tree diagram, makes sure no outcome is missed.

Key terms

- **Compound event:** an event made of two or more stages.
- **Sample space:** the set of all possible outcomes.
- **Tree diagram:** branches that show every choice at every stage.
- **Simulation:** a model that uses a random device to imitate a real chance process.

How does a tree diagram work?

Each branch is a choice. A coin has 2 branches, and a spinner with 3 equal sections adds 3 branches to each of them. Follow each path from left to right to read one outcome. There are $2 \times 3 = 6$ outcomes: H1, H2, H3, T1, T2, T3.



Tree diagram: coin, then spinner. Six outcomes.

How do you find the probability of a compound event?

When every outcome in the sample space is equally likely, count the outcomes that match the event and divide by the total.

Compound event probability

$$P(\text{event}) = \frac{(\text{number of outcomes in the event})}{(\text{number of outcomes in the sample space})}$$

Worked example

Find $P(\text{heads and an odd number})$ for the coin and spinner above. The matching outcomes are H1 and H3, which is 2 out of 6.

$$P = 2/6 = 1/3.$$

A table works well for two number cubes. The table has 36 cells, and 6 of them have a sum of 7, so $P(\text{sum of 7}) = 6/36 = 1/6$.

		Cube 2					
Cube 1	1	2	3	4	5	6	
1	2	3	4	5	6	7	
2	3	4	5	6	7	8	
3	4	5	6	7	8	9	
4	5	6	7	8	9	10	
5	6	7	8	9	10	11	
6	7	8	9	10	11	12	

Two number cubes: the 6 shaded cells have a sum of 7.

How do you design a simulation?

1. Pick a random device that matches the chance of each stage, such as random digits 0 to 9.
2. Decide what the outcomes mean.
3. Define one trial and a success.
4. Run many trials and count the successes.
5. Divide the successes by the number of trials.

To imitate a player who makes 70% of free throws, let digits 0 to 6 mean a made shot and 7 to 9 a miss. A trial is two digits, one for each shot.

0	1	2	3	4	5	6	7	8	9
0 to 6: shot made						7 to 9: miss			

A digit key for simulating a 70% free-throw shooter.

Ten trials gave 52, 81, 14, 69, 73, 07, 90, 36, 28, 55. Both digits are 6 or less in 52, 14, 36 and 55, so the experimental probability of two made shots is $4/10 = 0.4$. The theoretical value is $0.7 \times 0.7 = 0.49$.

Common mistake

Forgetting that order matters. Heads then tails (HT) and tails then heads (TH) are different outcomes, so both belong in the sample space.

Mathbot's tip

Count the total outcomes first by multiplying the number of choices at each stage, then check that your list has the same count.

Key points to remember

- Sample space = list of all outcomes; count with multiplication.
- $P(\text{event}) = \text{matching outcomes} \div \text{total outcomes}$.
- Tables suit two stages; tree diagrams suit two or more stages.
- Order matters: HT and TH are different outcomes.
- A simulation estimates a probability from many random trials.

Compound Events, Sample Spaces and Simulations: frequently asked questions

What is a compound event?

A compound event has two or more stages, such as flipping a coin and rolling a number cube. Its sample space contains one outcome for every combination of the stages.

How many outcomes are in a sample space?

Multiply the number of choices at each stage. Flipping a coin (2 choices) and rolling a number cube (6 choices) gives $2 \times 6 = 12$ outcomes.

When should I use a table and when a tree diagram?

A table is quick for two stages, like two number cubes. A tree diagram works for two or more stages and shows every path, such as three coin flips.

Why does a simulation give a different answer than the theoretical probability?

A simulation uses a limited number of random trials, so its result varies. More trials usually bring the result closer to the theoretical probability.

Does the order of the outcomes matter?

Yes. If the stages are different or happen in sequence, HT and TH are separate outcomes. Listing both prevents undercounting the sample space.