

# Building Triangles from Sides and Angles

Grade 7 math · Worksheet Answer Key · Standards: 7.G.A.2

Each answer shows the inequality check or angle sum that decides the case, and then the conclusion in bold. If your answer differs, check the equal-sum case, which is the easiest one to miss, and whether the angle was between the sides.

**Standards:** 7.G.A.2 · **Level:** Grade 7

## Problem 1 Three Poles, First Test

The two shorter sides add to  $5 + 6 = 11$ , which is less than 12. They cannot reach across.

**No triangle**

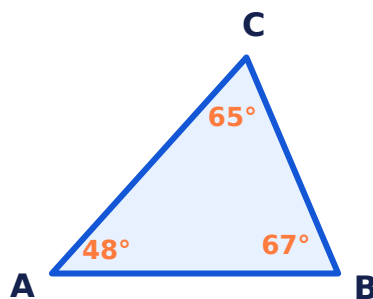
## Problem 2 Three Poles, Second Test

Check all pairs:  $7 + 9 = 16 > 12$ ,  $7 + 12 = 19 > 9$  and  $9 + 12 = 21 > 7$ . The longest-side check is enough.

**Yes, a triangle can be formed**

## Problem 3 Find the Third Angle

$48 + 67 = 115$ , and  $180 - 115 = 65$ .



*The three angles add to 180°.*

**65°**

#### Problem 4 Which Set Works

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(a)  $8 + 8 = 16 > 15$ : yes. (b)  $6 + 9 = 15 < 16$ : no. (c)  $10 + 10 = 20$ , not greater than 20: no.

**Only (a) forms a triangle**

#### Problem 5 Range of the Third Side

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The third side must be less than  $7 + 11 = 18$ , and it must be greater than  $11 - 7 = 4$  so that 7 and the new side reach across 11.

**Greater than 4 in and less than 18 in**

#### Problem 6 A Flat Triangle

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$6 + 9 = 15$ , which is equal to the third side, not greater. The two shorter sides lie exactly along the longest side, so the "triangle" collapses to a straight segment with no area.

**No triangle, only a flat segment**

#### Problem 7 Count the Triangles

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(a) Angles add to  $180^\circ$ , but no side is given, so infinitely many triangles fit. (b)  $70 + 80 + 50 = 200$ , not 180, so none. (c) SAS: exactly one. (d)  $4 + 4 = 8 < 9$ , so none.

**(a) many, (b) none, (c) exactly one, (d) none**

#### Problem 8 ASA Conditions

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Third angle:  $180 - 35 - 80 = 65$ . Two angles with the side between them (ASA) fix one triangle.

**$65^\circ$ ; unique**

#### Problem 9 Classify by Angles

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$112 + 41 = 153$ , and  $180 - 153 = 27$ . One angle is  $112^\circ$ , which is greater than  $90^\circ$ .

**$27^\circ$ ; obtuse**

#### Problem 10 Base of an Isosceles Triangle

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The base must be less than  $9 + 9 = 18$ , and it must be positive. Also  $9 + \text{base} > 9$  holds for any positive base.

**1, 2, 3, ..., 17 inches (any length from 0 to 18 exclusive)**

### Problem 11 Longest Side Known

Call the third side  $x$ . Since 14 is longest,  $x \leq 14$ . The two shorter sides must exceed 14:  $6 + x > 14$ , so  $x > 8$ .

**9, 10, 11, 12, 13 or 14 inches**

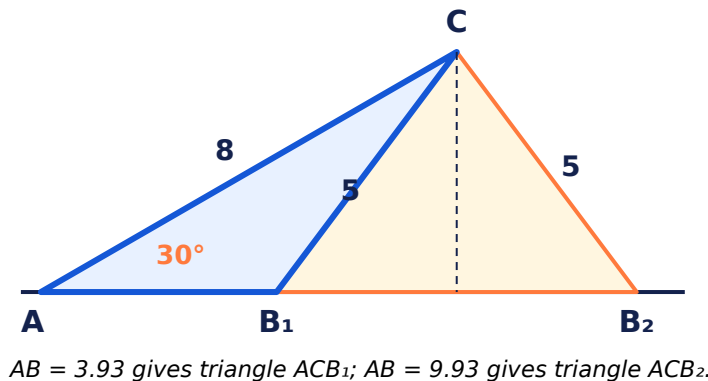
### Problem 12 Two Triangles from SSA

The side  $BC = 5$  is longer than the height 4 and shorter than  $AC = 8$ , so it can swing to two points on line  $AB$ .

Distance from the foot of the height to  $B$ :  $5^2 - 4^2 = 9$ , so 3 units. The foot is  $8 \cos 30^\circ = 4\sqrt{3} \approx 6.93$  from  $A$ .

$AB = 6.93 - 3 \approx 3.93$  or  $AB = 6.93 + 3 \approx 9.93$ .

**Two triangles,  $AB \approx 3.93$  and  $AB \approx 9.93$**



### Problem 13 Same Angles, Different Sizes

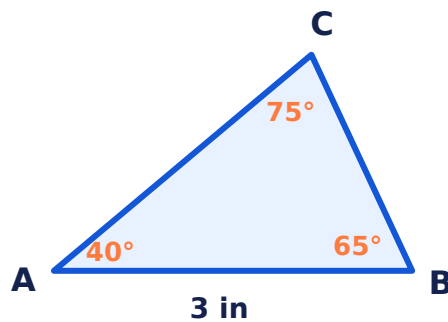
Each set is ASA, so each triangle is unique, but the given sides differ. The third angle is  $70^\circ$  in both. Zane's triangle is an enlargement of Maya's with scale factor  $8 \div 4 = 2$ .

**No, they have the same shape but not the same size**

### Problem 14 Draw and Compare

Angle  $C = 180 - 40 - 65 = 75^\circ$ . The longest side lies opposite the largest angle, which is  $C = 75^\circ$ , so the longest side is  $AB$ , the side of 3 in. The shortest is  $BC$ , opposite  $A = 40^\circ$ .

**Angle  $C = 75^\circ$ ;  $AB$  is the longest side**



*AB faces the largest angle.*

### Problem 15 Poles for a Tent Frame

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$9 + 12 = 21$ , which is not greater than 21. The poles would lie flat.

**No triangular frame is possible**

### Problem 16 Roof Truss Beams

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The beam must be greater than  $14 - 10 = 4$  and less than  $10 + 14 = 24$ . Whole numbers from 5 to 23:  $23 - 5 + 1 = 19$ .

**19 different lengths**

### Problem 17 Hiking Triangle

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The return leg is less than  $3 + 4.5 = 7.5$  and greater than  $4.5 - 3 = 1.5$ .

**Greater than 1.5 mi and less than 7.5 mi**

### Problem 18 Sail Angles

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$74 + 58 = 132$ , and  $180 - 132 = 48$ . All three angles ( $74^\circ$ ,  $58^\circ$ ,  $48^\circ$ ) are less than  $90^\circ$ .

**$48^\circ$ ; acute**

### Problem 19 Stained Glass Piece

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Third angle:  $180 - 40 - 85 = 55^\circ$ .

The conditions are ASA, so the triangle is unique. Steps: draw a 5 in segment, mark  $40^\circ$  at one end and  $85^\circ$  at the other with a protractor, draw both rays and mark where they meet.

**55°; unique**

### **Problem 20 Surveyor and the Tree**

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BC = 3: shorter than the height 5, so it cannot reach the line, giving none.

BC = 5: equal to the height, giving exactly one (a right triangle).

BC = 7: between 5 and 10, giving two triangles.

BC = 12: longer than AC = 10, giving exactly one.

**3 ft: none; 5 ft: one; 7 ft: two; 12 ft: one**