

Fractions, Decimals and Rational Number Word Problems

Grade 7 math · Lesson · Standards: 7.NS.A.2d, 7.NS.A.3

Quick answer: Every rational number can be written as a decimal by dividing the numerator by the denominator. The decimal either terminates, like $\frac{3}{8} = 0.375$, or repeats a block of digits forever, like $\frac{5}{6} = 0.8\bar{3}$. Word problems with rational numbers use the same sign rules as integers, applied to fractions and decimals.

Standards: 7.NS.A.2d, 7.NS.A.3 · **Level:** Grade 7

How do you change a fraction into a decimal?

A fraction is a division problem in disguise: the fraction bar means "divided by." To convert, divide the numerator by the denominator with long division, attaching zeros after the decimal point whenever you run out of digits.

Terminating and repeating decimals

Every rational number is a decimal that either **terminates** (a remainder of 0 appears) or **repeats** (a block of digits cycles forever). A bar marks the repeating block: $0.8333... = 0.8\bar{3}$.

$$\begin{array}{r}
 \text{0 . 3 7 5} \\
 8 \overline{) 3 . 0 0 0} \\
 \underline{24} \\
 60 \\
 \underline{56} \\
 40 \\
 \underline{40} \\
 0
 \end{array}$$

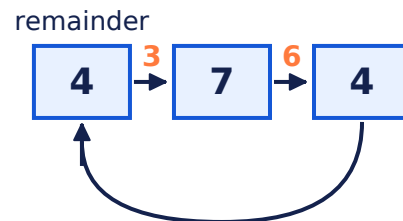
Long division of 3 by 8: the remainder reaches 0, so the decimal terminates at 0.375.

How can you predict whether a decimal stops or repeats?

Write the fraction in lowest terms and factor the denominator. If the only prime factors are 2 and 5,

the decimal terminates. Any other prime factor forces a repeat, because only a limited number of remainders exist, so one must eventually return.

Fraction	Denominator factors	Decimal	Type
$\frac{7}{20}$	$2 \cdot 2 \cdot 5$	0.35	terminates
$\frac{5}{6}$	$2 \cdot 3$	$0.8\overline{3}$	repeats
$\frac{4}{11}$	11	$0.3\overline{6}$	repeats



remainders repeat, so the digits repeat

Dividing 4 by 11: remainder 4 leads to remainder 7, which leads back to remainder 4, so the digits 3 and 6 repeat.

Reduce first

$\frac{9}{12}$ has a 3 hiding in the denominator, yet in lowest terms it is $\frac{3}{4} = 0.75$, which terminates.
Always simplify before you factor.

How do you solve real-world problems with rational numbers?

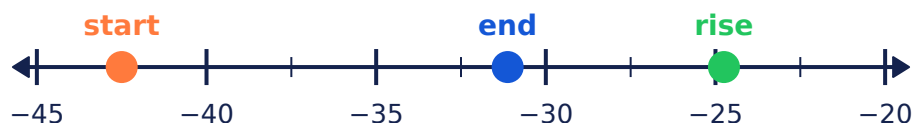
Method for rational word problems

1. Choose one form, all decimals or all fractions, and write each phrase as an addition, subtraction, multiplication or division.
2. Apply the sign rules: same signs add the sizes, different signs subtract the smaller size from the larger and keep the sign of the larger.
3. Compute, then compare with a quick estimate.
4. Answer in a sentence with units.

Worked example

A diver at -42.5 ft rises 17.75 ft, then sinks $6\frac{3}{8}$ ft. Where is the diver now?

$$6\frac{3}{8} = 6.375, \text{ so } -42.5 + 17.75 - 6.375 = -24.75 - 6.375 = \mathbf{-31.125 \text{ ft}}$$



The diver starts at -42.5 ft, rises to -24.75 ft, then sinks to -31.125 ft.

Mathbot's tip

Estimate first: $-42 + 18 - 6$ is about -30 , so -31.125 passes the sniff test.

Key points to remember

- A fraction means numerator $\div$ denominator, so long division converts it to a decimal.
- A decimal terminates when the reduced denominator has only the prime factors 2 and 5.
- All other reduced denominators give repeating decimals, marked with a bar over the repeating block.
- Subtracting a negative number is the same as adding its opposite, for fractions and decimals alike.

Fractions, Decimals and Rational Number Word Problems: frequently asked questions

Is a repeating decimal a rational number?

Yes. Any decimal that terminates or repeats equals a fraction of two integers, so it is rational. Decimals that never repeat and never end, such as π , are not rational.

Why does long division of a fraction always end or repeat?

Each step leaves a remainder smaller than the divisor, so only a limited list of remainders is possible. A remainder of 0 ends the decimal, and a repeated remainder starts the cycle again.

Is 0.999... equal to 1?

Yes. Since $1/3 = 0.333\dots$, multiplying both sides by 3 gives $1 = 0.999\dots$. The two names describe the same point on the number line.

Should I use fractions or decimals in a word problem?

Use whichever form makes the arithmetic easier and keep it consistent. Fractions with repeating decimals, like $1/3$, are usually safer as fractions until the final step.

How do I know the sign of a product or quotient?

Same signs give a positive result and different signs give a negative result. Count the negative factors: an even count is positive, an odd count is negative.