

# Finding Unit Rates When Quantities Are Fractions

Grade 7 math · Lesson · Standards: 7.RP.A.1

**Quick answer:** A unit rate with fractions tells how much of one quantity matches exactly 1 unit of another when the measurements are fractions. Divide the first quantity by the second, multiply by the reciprocal of the divisor, and simplify. For example, 1/2 mile in 1/4 hour is 2 miles per hour.

**Standards:** 7.RP.A.1 · **Level:** Grade 7

## What is a unit rate when the measurements are fractions?

A unit rate says how much of one quantity matches exactly 1 unit of another quantity. When the measurements themselves are fractions, the rate is written as a *complex fraction*, a fraction whose top and bottom are fractions.

### Unit rate

A unit rate compares two quantities so that the second quantity equals 1. Examples: miles per hour, dollars per pound, cups per batch, square feet per minute.

## How do you simplify a complex fraction?

A fraction bar means division. So a complex fraction is a division of two fractions, and dividing by a fraction means multiplying by its reciprocal.

### Rate formula

$$\frac{a/b}{c/d} = \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

The quantity on top of the fraction bar is the one being measured; the quantity on the bottom is the one that becomes 1.



### Method: finding a unit rate from fractions

1. Decide which quantity is measured and which one should become 1 (the word after "per").
2. Write the quantities as a division, first quantity  $\div$  second quantity.
3. Change any mixed numbers to improper fractions.
4. Multiply by the reciprocal of the second fraction.
5. Simplify and write the answer with its unit.

### Worked example

A kayaker paddles  $\frac{5}{6}$  mile in  $\frac{3}{8}$  hour. Find the speed in miles per hour.

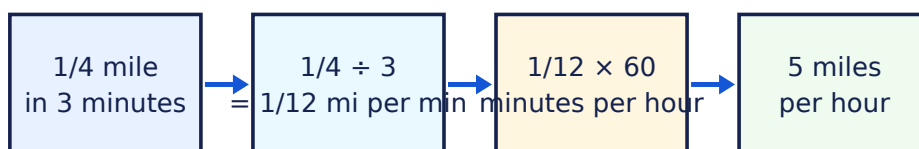
$\frac{5}{6} \div \frac{3}{8} = \frac{5}{6} \times \frac{8}{3} = \frac{40}{18} = \frac{20}{9} = 2 \frac{2}{9}$ . The unit rate is  **$2 \frac{2}{9}$  miles per hour**.

|                  |            |          |
|------------------|------------|----------|
| Time (hours)     | <b>3/8</b> | <b>1</b> |
| Distance (miles) | 5/6        | 20/9     |

Both rows are multiplied by  $\frac{8}{3}$ , so the time becomes exactly 1 hour.

### What if the two quantities use different units?

Compute the rate in the units you are given, then convert. A rate in miles per minute becomes miles per hour when multiplied by 60, because 1 hour has 60 minutes.



Different units: find the rate per minute first, then convert minutes to hours.

### Common mistake

Dividing in the wrong order. For  $\frac{1}{4}$  mile in 3 minutes,  $\frac{1}{4} \div 3$  gives miles per minute, but  $3 \div \frac{1}{4}$  gives minutes per mile. Say the rate aloud, "miles per minute", to pick the order.

### Mathbot's tip

Check your answer by multiplying it back: unit rate  $\times$  second quantity should return the first quantity. If  $2 \frac{2}{9} \times \frac{3}{8}$  gives  $\frac{5}{6}$ , you are right.

### Key points to remember

- A unit rate has 1 in the denominator: miles per 1 hour, dollars per 1 pound.
- A complex fraction is just a division of two fractions.
- Dividing by a fraction means multiplying by its reciprocal.
- When units differ, convert after finding the rate (for example, multiply by 60 to go from minutes to hours).
- Multiplying the unit rate by the second quantity must give back the first quantity.

## **Finding Unit Rates When Quantities Are Fractions: frequently asked questions**

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### **What is a complex fraction?**

A complex fraction has a fraction in its numerator, its denominator, or both. It is read as a division: the top fraction divided by the bottom fraction.

### **Why do we multiply by the reciprocal?**

Dividing by a fraction asks how many of that fraction fit into the amount. Multiplying by the reciprocal gives the same result and is faster, because the reciprocal of  $\frac{3}{8}$  is  $\frac{8}{3}$ .

### **How do I know which quantity to divide by?**

Look at the word after per. In miles per hour, hours is the quantity that becomes 1, so divide miles by hours.

### **Can a unit rate be a fraction or a mixed number?**

Yes. A unit rate can be any positive rational number, such as  $2\frac{2}{9}$  miles per hour or  $\frac{3}{4}$  cup per batch.

### **How do I find a unit rate when the units are different?**

Divide the quantities as given, then convert with a conversion factor such as 60 minutes per hour or 3 feet per yard. Check that the final unit matches the question.