

Understanding Functions and Their Graphs

Grade 8 math · Lesson · Standards: 8.F.A.1, 8.F.A.2, 8.F.A.3

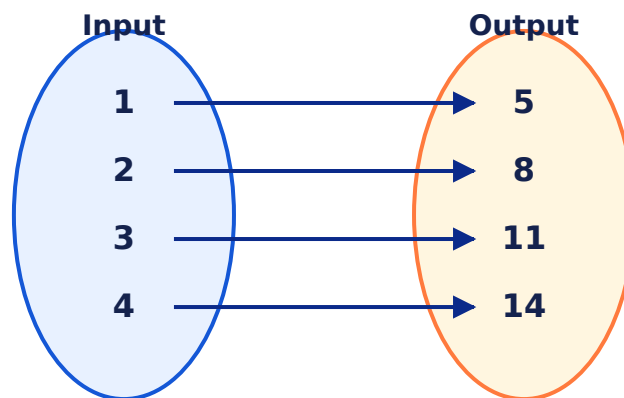
Quick answer: A function assigns exactly one output to every input. You can check it in a table, a mapping, a graph (vertical line test) or an equation. Linear functions, $y = mx + b$, change at a constant rate and graph as straight lines; nonlinear functions such as $A = s^2$ do not.

Standards: 8.F.A.1, 8.F.A.2, 8.F.A.3 · **Level:** Grade 8

What is a function?

Function

A function is a rule that assigns exactly one output to each input. If an input could produce two different outputs, the relation is not a function.

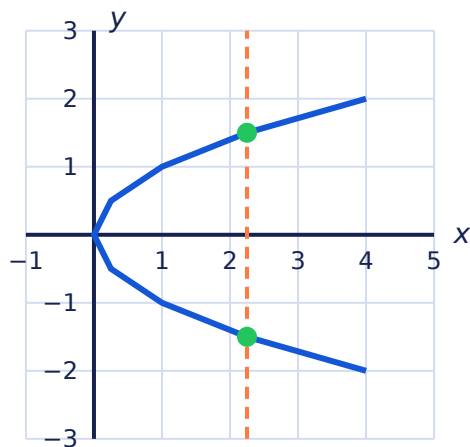


Each input has exactly one arrow, so this is a function: $\text{output} = 3 \cdot \text{input} + 2$.

We write $f(x)$ for the output of the function f at input x . For $f(x) = 3x + 2$, we have $f(4) = 14$.

How do you test whether a relation is a function?

Representation	Check
Ordered pairs or table	No input appears with two different outputs
Mapping diagram	Every input has exactly one arrow
Graph	Vertical line test: no vertical line crosses the graph twice
Equation	Each value of x gives only one value of y



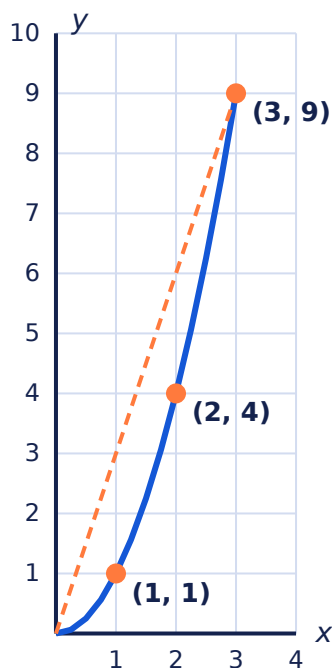
A vertical line hits this curve twice, so it is not the graph of a function.

Two inputs may share an output. A function can send 2 and -2 to the same value, but it cannot send one input to two values.

How do you tell linear functions from nonlinear ones?

Linear function

A linear function has the form $y = mx + b$. Its rate of change m is constant, so its graph is a straight line. If the rate of change varies, the function is nonlinear.



Area $A = s^2$ bends away from the straight line, so it is nonlinear.

For the area of a square, the values 1, 4, 9 grow by 3, then by 5. The rate is not constant, so $A = s^2$ is nonlinear.

Method: compare two functions

1. Get the rate of change and initial value of each from its table, graph or equation.
2. Compare rates: the larger rate grows faster.
3. Compare initial values: the value when $x = 0$.

Example

Function A: $y = 5x + 2$. Function B: a table with (0, 9) and (2, 17). B has rate $(17 - 9) \div 2 = 4$ and initial value 9.

A grows faster ($5 > 4$), but B starts higher ($9 > 2$).

Common mistake

Thinking repeated outputs break the rule. The rule only forbids one input having two outputs, so $f(2) = f(-2) = 4$ is fine.

Mathbot's tip

Beep! In a table, check the differences. If equal steps in x give equal steps in y , the function is linear.

Key points to remember

- Function: every input has exactly one output.
- Vertical line test: a graph is a function if no vertical line hits it twice.
- Different inputs may share the same output.
- Linear function: $y = mx + b$ with constant rate of change m .
- Compare functions by rate of change and initial value, whatever their representation.

Understanding Functions and Their Graphs: frequently asked questions

What is the difference between a relation and a function?

A relation is any set of input-output pairs. A function is a relation where each input has exactly one output, so $\{(1, 2), (1, 5)\}$ is not a function.

How does the vertical line test work?

Slide a vertical line across the graph. If it never touches the graph in more than one point, each input has one output and the graph is a function.

Can two different inputs have the same output?

Yes. For $f(x) = x^2$, both 3 and -3 give 9. That is allowed, because the rule only restricts one input from having two outputs.

How can you tell if a function is linear from a table?

Compute the change in y for equal steps in x . If the rate of change is constant, the function is linear; if it varies, it is nonlinear.

What does $f(5)$ mean?

It means the output of the function f when the input is 5. For $f(x) = 4x - 7$, substitute 5 for x to get $f(5) = 13$.