

Rational and Irrational Numbers Explained

Grade 8 math · Lesson · Standards: 8.NS.A.1, 8.NS.A.2

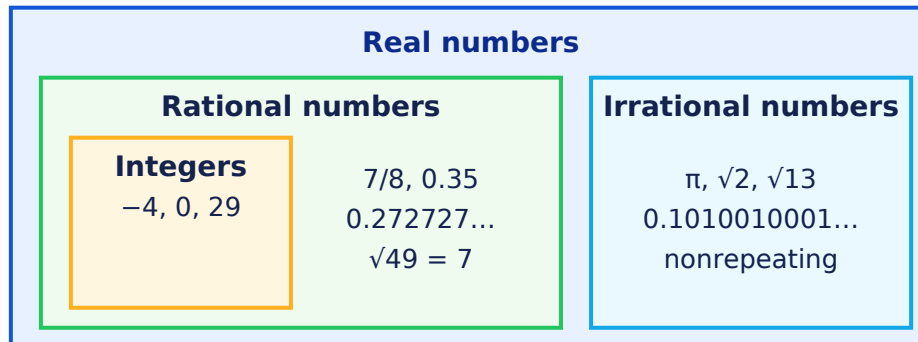
Quick answer: Rational numbers are quotients of integers, so their decimals end or loop. Irrational numbers such as $\sqrt{2}$ and π have endless decimals with no loop. Put the two families together and you get every real number, with no overlap between them: each number belongs to just one family.

Standards: 8.NS.A.1, 8.NS.A.2 · **Level:** Grade 8

What makes a number rational or irrational?

Rational and irrational numbers

A **rational number** is a quotient $\frac{a}{b}$ of two integers with $b \neq 0$; written as a decimal, it ends or loops through one block. An **irrational number** has no such quotient, and its digits wander on without a loop.



No real number belongs to both groups.

Square roots of perfect squares, such as $\sqrt{169} = 13$, are rational. Square roots of other whole numbers, such as $\sqrt{35}$, are irrational.

How can a repeating decimal be rewritten as a fraction?

Method: repeating decimal to fraction

1. Name the number x and mark the repeating block.
2. Shift the decimal point past one full block: multiply by 10, 100 or 1,000 for blocks of one, two or three digits.
3. Subtract x from the shifted equation so the loop disappears.
4. Solve for x and simplify.

Example: $x = 0.318318318\dots$

The block 318 has three digits, so multiply by 1,000: $1,000x = 318.318318\dots$

Subtract: $1,000x - x = 318$, so $999x = 318$.

Then $x = \frac{318}{999} = \frac{106}{333}$.

How can you approximate a root that is not a whole number?

Trap the root between two perfect squares, then refine with tenths. Since $49 < 54 < 64$, we know $7 < \sqrt{54} < 8$. Testing $7.3^2 = 53.29$ and $7.4^2 = 54.76$ gives $7.3 < \sqrt{54} < 7.4$.



$\sqrt{54}$ sits between 7.3 and 7.4 because $7.3^2 = 53.29$ and $7.4^2 = 54.76$.

Number	Decimal form	Type
$\frac{5}{16}$	0.3125 (stops)	Rational
$\frac{4}{15}$	0.2666... (repeats)	Rational
$\sqrt{26}$	5.0990195... (no pattern)	Irrational

Which of two irrational numbers is larger?

Write each number as a decimal, or square both numbers when roots are involved. Compare $\pi \approx 3.1416$ with $\sqrt{10} \approx 3.1623$: the first digits agree until the hundredths place, and $3.14 < 3.16$, so $\pi < \sqrt{10}$.



Squaring 3.1623 gives about 10.00, and $\pi \approx 3.1416$, so π is the smaller number.

Common mistake

A rounded decimal such as 3.14 is rational, but π itself is not. An approximation never changes the type of the original number.

Mathbot's tip

Beep! To place $\sqrt{n}$, find the perfect squares on each side of n . The root lives between their roots, and it sits closer to the nearer square.

Key points to remember

- Rational: a fraction of integers; decimal stops or repeats.
- Irrational: decimal never stops and never repeats (π , $\sqrt{2}$, $\sqrt{35}$).
- The square root of a non-perfect square is irrational.
- Multiply a repeating decimal by 10, 100 or 1,000, then subtract to find its fraction.
- Estimate $\sqrt{n}$ by trapping it between two perfect squares.

Rational and Irrational Numbers Explained: frequently asked questions

Is zero a rational number?

Yes. Zero equals $0/1$, a fraction of two integers, so it is rational. Every integer is rational because it can be written over 1.

Is a repeating decimal rational?

Yes. Every repeating decimal equals a fraction. For example, $0.8888\dots$ equals $8/9$, which you can prove by multiplying by 10 and subtracting.

Is $\sqrt{2}$ rational or irrational?

It is irrational. Its decimal begins $1.41421356\dots$ and never ends or repeats, and no fraction of integers equals it.

Can an irrational number be written exactly as a decimal?

No. A decimal for an irrational number never ends, so any written decimal is only an approximation. The symbol, such as $\sqrt{7}$ or π , is the exact value.

How do you know which of two square roots is larger?

Compare the numbers under the radical signs. Since 29 is greater than 23, $\sqrt{29}$ is greater than $\sqrt{23}$. For a root versus a decimal, square the decimal and compare.