

Rational and Irrational Numbers Explained

Grade 8 math · Worksheet Answer Key · Standards: 8.NS.A.1, 8.NS.A.2

Each answer shows the steps in full: the perfect squares tested, the subtraction that removes a repeating tail, and the final result in bold. Compare each line with your own work, and rework any problem whose steps differ.

Standards: 8.NS.A.1, 8.NS.A.2 · **Level:** Grade 8

Problem 1 Classify the numbers

$0.75 = \frac{3}{4}$ is rational. $\sqrt{81} = 9$ is rational. $\sqrt{35}$ is not a perfect square, so it is irrational. $-\frac{7}{9}$ is rational. π is irrational. $0.121212\dots$ repeats, so it is rational.

Irrational: $\sqrt{35}$ and π . The other four are rational.

Problem 2 Perfect squares and roots

$\sqrt{16} = 4$, $\sqrt{144} = 12$ and $\sqrt{0.25} = 0.5$ are whole numbers or terminating decimals. $\sqrt{50}$ and $\sqrt{6}$ lie between consecutive integers and are not perfect squares.

Rational: $\sqrt{16}$, $\sqrt{144}$, $\sqrt{0.25}$. Irrational: $\sqrt{50}$, $\sqrt{6}$.

Problem 3 Terminating decimals as fractions

(a) $0.375 = \frac{375}{1,000}$. Divide top and bottom by 125 to get $\frac{3}{8}$.

(b) $2.6 = \frac{26}{10} = \frac{13}{5}$.

(a) $\frac{3}{8}$ (b) $\frac{13}{5}$

Problem 4 Single-digit repeating decimals

A one-digit block over 9 works. (a) Let $x = 0.888\dots$; $10x - x = 8$, so $x = \frac{8}{9}$.

(b) $10x - x = 2$, so $x = \frac{2}{9}$.

(a) $\frac{8}{9}$ (b) $\frac{2}{9}$

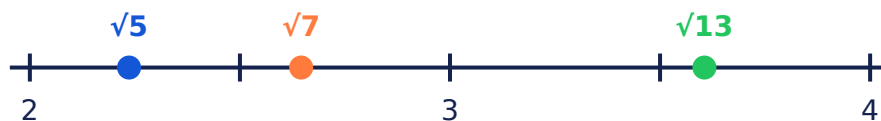
Problem 10 Square root to the nearest tenth

$5.2^2 = 27.04$ and $5.3^2 = 28.09$. Since 28 is much closer to 28.09, $\sqrt{28}$ is close to 5.3. Check $5.29^2 = 27.9841$, so $\sqrt{28} \approx 5.29$, which rounds to 5.3.

$$\sqrt{28} \approx 5.3$$

Problem 11 Match roots to points

Squares: $2.2^2 = 4.84$, $2.3^2 = 5.29$, so $\sqrt{5} \approx 2.24$. $2.6^2 = 6.76$, $2.7^2 = 7.29$, so $\sqrt{7} \approx 2.65$. $3.6^2 = 12.96$, so $\sqrt{13} \approx 3.61$. Order the points left to right.



Labeled points.

$$A = \sqrt{5}, B = \sqrt{7}, C = \sqrt{13}$$

Problem 12 Root versus decimal

(a) $5.5^2 = 30.25$, which is more than 30, so $\sqrt{30} < 5.5$. (b) $7^2 = 49$, which is less than 50, so $\sqrt{50} > 7$.

$$(a) \sqrt{30} < 5.5 \quad (b) \sqrt{50} > 7$$

Problem 13 Order from least to greatest

$\sqrt{17}$: $4.1^2 = 16.81$ and $4.2^2 = 17.64$, so $\sqrt{17} \approx 4.123$. $1 + \pi \approx 4.1416$. $4\frac{1}{8} = 4.125$. And 4.2 is exact.

$$4.123 < 4.125 < 4.1416 < 4.2.$$

$$\sqrt{17}, 4\frac{1}{8}, 1 + \pi, 4.2$$

Problem 14 Rational or irrational results

(a) Adding a rational number to $\sqrt{2}$ leaves a never-ending, nonrepeating decimal: irrational. (b) $\sqrt{2} \cdot \sqrt{2} = 2$, which is rational. (c) A nonzero rational times π stays irrational.

$$(a) \text{ irrational } (b) \text{ rational } (c) \text{ irrational}$$

Problem 15 Square garden

Side = $\sqrt{75}$. Since $8^2 = 64$ and $9^2 = 81$, the side is between 8 and 9 feet. $8.6^2 = 73.96$ and $8.7^2 = 75.69$, and 75 is closer to 75.69 than to 73.96. Check $8.66^2 = 74.9956$.

About 8.7 feet

Problem 16 Lap fraction

Let $x = 0.6363\dots$. Multiply by 100: $100x = 63.6363\dots$. Subtract: $99x = 63$, so $x = \frac{63}{99} = \frac{7}{11}$.

$\frac{7}{11}$ mile

Problem 17 Is 0.7272... the same as 0.72?

$0.7272\dots = \frac{72}{99} = \frac{8}{11}$. And $0.72 = \frac{72}{100} = \frac{18}{25}$. Common denominator 275: $\frac{8}{11} = \frac{200}{275}$ and $\frac{18}{25} = \frac{198}{275}$.

Since $200 > 198$, the repeating decimal is larger. **Ravi is wrong; $0.7272\dots > 0.72$.**

Problem 18 Tile side length

$12^2 = 144$ and $13^2 = 169$, so the side is between 12 and 13. Then $12.2^2 = 148.84$ and $12.3^2 = 151.29$. Since 150 is between these, $\sqrt{150}$ is between 12.2 and 12.3. The midpoint $12.25^2 = 150.0625$ is above 150, so $\sqrt{150} < 12.25$ and rounds down.

About 12.2 inches

Problem 19 Is 3.14 equal to π ?

3.14 is a terminating decimal, so it is rational. π is irrational, so it cannot equal any terminating or repeating decimal. Using $\pi = 3.14159\dots$, we get $3.14 < \pi < 3.15$.

3.14 is only an approximation; $3.14 < \pi < 3.15$.

Problem 20 Close call

$6.7^2 = 44.89$, which is less than 45, so $\sqrt{45} > 6.7$. Then $6.71^2 = 45.0241$ is more than 45, so $6.70 < \sqrt{45} < 6.71$. The difference is less than 0.01 and about 0.008, which rounds to 0.01.

$\sqrt{45}$ is greater, by about 0.01.