

Modeling Real Situations With Linear Functions

Grade 8 math · Lesson · Standards: 8.F.B.4, 8.F.B.5

Quick answer: A linear function $y = mx + b$ has a constant rate of change m and an initial value b . Find m from two points as change in y over change in x , then find b by substituting a point. In context, m is the change per unit and b is the starting amount.

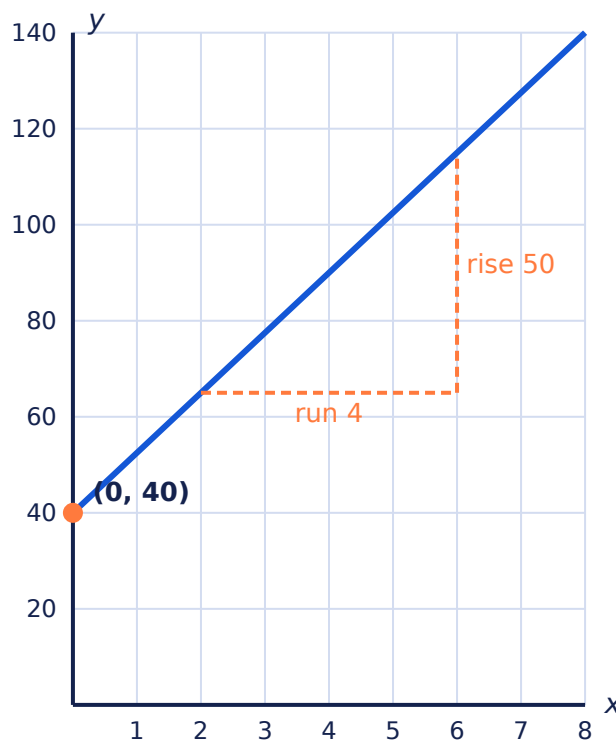
Standards: 8.F.B.4, 8.F.B.5 · **Level:** Grade 8

How do you build a linear function from a description?

Linear function

A linear function is $y = mx + b$. The **rate of change** m is how much y changes per 1 unit of x . The **initial value** b is the output when $x = 0$.

A hiking club fund starts at \$40 and grows by \$12.50 each week. The rate is 12.5 and the initial value is 40, so $y = 12.5x + 40$.



Fund balance: starts at \$40 and grows \$12.50 per week, so $y = 12.5x + 40$.

How do you find the function from two points, a table or a graph?

Method

1. Find the rate: $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$. In a table or graph, use any two points.
2. Find the initial value: substitute one point into $y = mx + b$ and solve for b .
3. Write $y = mx + b$ and test it against a third point.

Example: through (2, 17) and (6, 29)

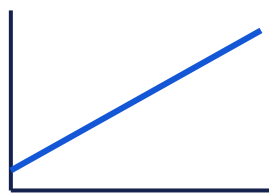
Rate: $(29 - 17) \div (6 - 2) = 3$. Initial value: $17 = 3(2) + b$, so $b = 11$.

The function is **$y = 3x + 11$** . Check (6, 29): $3(6) + 11 = 29$.

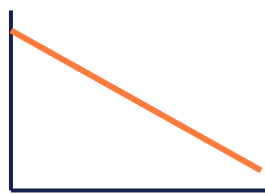
What do the rate and the initial value mean in context?

Model	Rate of change	Initial value
$C = 0.45m + 28$ (van rental)	\$0.45 per mile	\$28 base fee
$h = 120 - 8t$ (draining tank)	-8 gallons per minute	120 gallons at the start

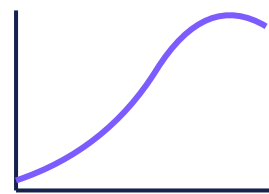
How do you describe a graph and sketch one?



increasing, linear



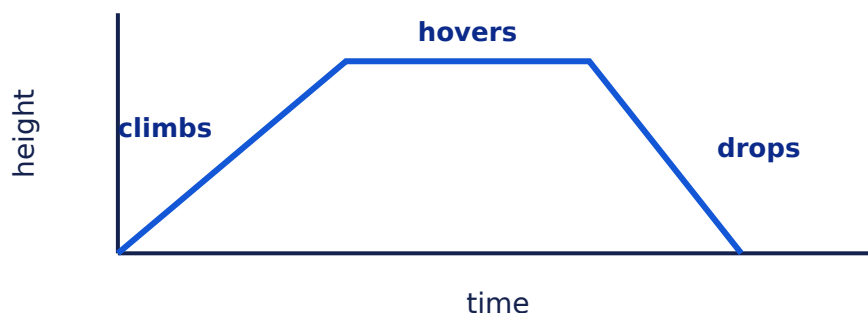
decreasing, linear



increasing, nonlinear

Describe a graph by direction (increasing or decreasing) and by shape (linear or nonlinear).

A function is **increasing** when the graph rises left to right, **decreasing** when it falls, and **constant** when it is level. It is linear if the graph is a straight line.



A sketch needs no numbers: it shows where the graph rises, stays level or falls.

Common mistake

Using the y-value of the first point as the rate. The rate is a change divided by a change, never a single output.

Mathbot's tip

Beep! A decreasing linear function has a negative rate. Write the minus sign: a tank draining 8 gallons per minute has rate -8 .

Key points to remember

- Rate of change $m = (y_2 - y_1) \div (x_2 - x_1)$.
- Initial value b is the output when $x = 0$.
- Find b by substituting a known point into $y = mx + b$.
- Increasing means the graph rises; decreasing means it falls; constant means level.
- A linear graph is a straight line; a nonlinear graph bends.

Modeling Real Situations With Linear Functions: frequently asked questions

How do you find the initial value of a linear function?

Substitute the rate and one known point into $y = mx + b$ and solve for b . For example, with $m = 3$ and the point $(2, 17)$, $17 = 6 + b$ gives $b = 11$.

What does the rate of change tell you in a real situation?

It says how much the output changes for each 1 unit of input, such as dollars per week or gallons per minute. A negative rate means the output is decreasing.

Can a linear function be decreasing?

Yes. When the rate of change m is negative, the function falls from left to right. A draining tank with rate -5 gallons per minute is an example.

How can you tell from a table if a function is linear?

Check that equal steps in x give equal changes in y . If the changes in y are constant, the function is linear; if they grow or shrink, it is nonlinear.

What is a qualitative graph?

It shows the general shape of a relationship, such as rising, level or falling, without exact numbers. You can sketch one from a story by marking each stage.