

Angle Relationships in Triangles and Parallel Lines

Grade 8 math · Lesson · Standards: 8.G.A.5

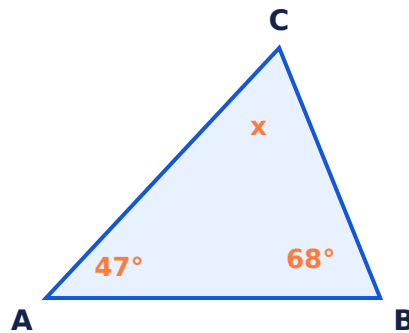
Quick answer: The three interior angles of any triangle add to 180° , and an exterior angle equals the sum of the two remote interior angles. When parallel lines are cut by a transversal, corresponding, alternate interior, and alternate exterior angles are equal, while same-side interior angles add to 180° .

Standards: 8.G.A.5 · **Level:** Grade 8

What is the angle sum of a triangle?

Triangle angle sum

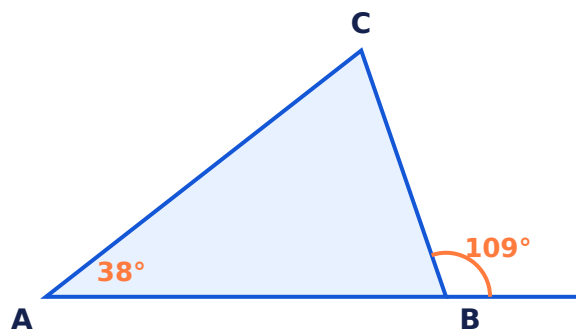
Add the three interior angles of any triangle and the total is always 180° . With two angles known, take their sum away from 180° to get the third.



Interior angles of a triangle: $47^\circ + 68^\circ + x = 180^\circ$, so $x = 65^\circ$.

What is an exterior angle?

Stretch one side of a triangle past a vertex, and the new angle outside the triangle is an exterior angle. The two interior angles that do not touch it are its remote interior angles.



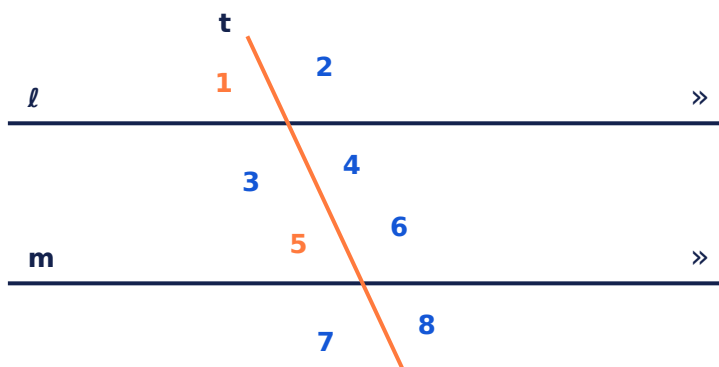
Exterior angle at B: $109^\circ = 38^\circ + 71^\circ$.

Exterior angle theorem

Each exterior angle is as large as its two remote interior angles put together. In the figure, $38^\circ + 71^\circ = 109^\circ$.

Which angles are equal when parallel lines are cut by a transversal?

A transversal is a line that cuts across two other lines at different points. If those two lines are parallel, the eight angles come in just two sizes. The acute ones match each other, the obtuse ones match each other, and any acute-obtuse pair makes a straight angle of 180° .



Parallel lines l and m with transversal t : $\angle 1, \angle 4, \angle 5, \angle 8$ are 65° ; $\angle 2, \angle 3, \angle 6, \angle 7$ are 115° .

Angle pairs for parallel lines

Pair of angles	Example	Relationship
Corresponding	$\angle 1$ and $\angle 5$	Equal (65°)
Alternate interior	$\angle 4$ and $\angle 5$	Equal (65°)
Alternate exterior	$\angle 1$ and $\angle 8$	Equal (65°)
Same-side interior	$\angle 3$ and $\angle 5$	Add to 180° ($115^\circ + 65^\circ$)
Vertical	$\angle 1$ and $\angle 4$	Equal (65°)

How do you use these facts?

Method

1. Sketch the lines and write each given angle on the sketch.
2. Name the fact that links the angles: vertical, corresponding, alternate, same-side, or triangle sum.
3. Turn that fact into an equation with x for the unknown.
4. Solve and check that angles in a triangle total 180° .

Worked example

Two parallel lines are cut by a transversal. One angle is $(3x + 8)^\circ$ and its corresponding angle is $(5x - 22)^\circ$. Corresponding angles are congruent, so $3x + 8 = 5x - 22$. Then $30 = 2x$, so $x = 15$ and each angle is 53° .

Common mistake

Same-side interior angles are not equal; they are supplementary. Check whether the angles look acute or obtuse before choosing between "equal" and "add to 180° ".

Mathbot's tip

Two angles that look different in size always add to 180° ; two that look the same size are equal. Let the picture choose the equation.

Key points to remember

- Interior angles of a triangle total 180° .
- An exterior angle equals the sum of the two remote interior angles.
- Parallel lines cut by a transversal make equal corresponding and alternate angles.
- Same-side interior angles are supplementary (sum 180°).
- Two triangles with two pairs of equal angles are similar (AA).

Angle Relationships in Triangles and Parallel Lines: frequently asked questions

Why do the angles of a triangle add to 180° ?

Draw a line through one vertex parallel to the opposite side. The alternate interior angles formed match the other two triangle angles, and the three angles along the straight line make 180° .

How can you tell alternate interior angles from corresponding angles?

Alternate interior angles lie between the parallel lines on opposite sides of the transversal. Corresponding angles sit in the same position at each intersection, such as both upper left.

What is the AA criterion for similar triangles?

If two angles of one triangle equal two angles of another, the third angles also match, so the triangles are similar. Their sides are then proportional.

Are vertical angles always equal?

Yes. Two crossing lines form two pairs of vertical angles, and each pair is equal. This is true whether or not the lines are parallel to anything else.

If corresponding angles are equal, are the lines parallel?

Yes. The converse also holds: when a transversal makes equal corresponding angles, or equal alternate interior angles, the two lines must be parallel.