

The Pythagorean Theorem and Its Converse

Grade 8 math · Lesson · Standards: 8.G.B.6, 8.G.B.7

Quick answer: The Pythagorean Theorem says that in a right triangle the squares of the two legs add up to the square of the hypotenuse: $a^2 + b^2 = c^2$. Its converse also holds, so if three side lengths satisfy this equation, the triangle has a right angle. It finds missing lengths in 2D and 3D.

Standards: 8.G.B.6, 8.G.B.7 · **Level:** Grade 8

What does the Pythagorean Theorem say?

Pythagorean Theorem

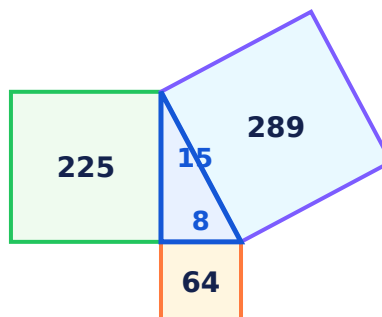
Name the legs a and b and the hypotenuse c in any right triangle. Then:

$$a^2 + b^2 = c^2$$

Side c faces the right angle and is the longest of the three.

Why is the theorem true?

Draw a square on every side of the triangle. The squares on the two legs, put together, have the same area as the one on the hypotenuse. One well-known proof fits four copies of the triangle into a large square in two different layouts. What remains uncovered is $a^2 + b^2$ in the first layout and c^2 in the second, so the two are equal.



Squares on the sides: $64 + 225 = 289$, so $8^2 + 15^2 = 17^2$.

How do you use the converse?

Converse

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Take any triangle and let c be its longest side. When $a^2 + b^2 = c^2$ is true, the angle opposite c is a right angle. When it is false, the triangle has no right angle.

Test 20, 21, 29: $20^2 + 21^2 = 400 + 441 = 841$, and $29^2 = 841$. The triangle is a right triangle.

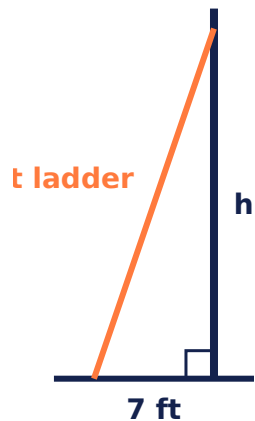
How do you find a missing side?

Method

1. Pick out the hypotenuse, the side across from the right angle, and call it c .
2. Substitute the known lengths into $a^2 + b^2 = c^2$.
3. Isolate the unknown squared length and take its positive square root.
4. Round when the root is not a whole number, and write the units.

Worked example

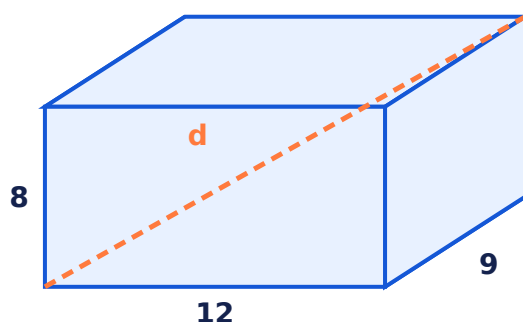
A 25-foot ladder has its base 7 feet from a wall. How high does it reach? The ladder is the hypotenuse, so $7^2 + h^2 = 25^2$. Then $h^2 = 625 - 49 = 576$ and $h = 24$ feet.



Ladder problem: $7^2 + h^2 = 25^2$.

How does it work in three dimensions?

Apply the theorem twice. For a box with length l , width w , and height h , the diagonal of the base is $\sqrt{l^2 + w^2}$, and the space diagonal d satisfies $d^2 = l^2 + w^2 + h^2$. For 12, 9, and 8: $144 + 81 + 64 = 289$, so $d = 17$.



Space diagonal of a box: $d^2 = 12^2 + 9^2 + 8^2$.

Common mistake

Taking the root of each term separately gives a wrong answer, because $\sqrt{a^2 + b^2}$ is not $a + b$. Add the squares, then take a single root.

Mathbot's tip

Ask first: is the unknown the longest side? If so, add the squares. If it is a leg, subtract.

Key points to remember

- In a right triangle, $a^2 + b^2 = c^2$, where c is the hypotenuse.
- The hypotenuse is opposite the right angle and is the longest side.
- If $a^2 + b^2 = c^2$ holds for a triangle, the triangle is right (converse).
- To find a leg, subtract: $a^2 = c^2 - b^2$.
- The space diagonal of a box satisfies $d^2 = l^2 + w^2 + h^2$.

The Pythagorean Theorem and Its Converse: frequently asked questions

Which side is the hypotenuse?

The hypotenuse is the side opposite the right angle, and it is always the longest side of a right triangle. In the equation it is the one that stands alone on the right side.

Can I use the theorem on any triangle?

No. It holds only for right triangles. For other triangles the sum of the squares of two sides does not match the square of the third side, which is exactly how the converse detects them.

What is a Pythagorean triple?

A Pythagorean triple is a set of three whole numbers that satisfy $a^2 + b^2 = c^2$, such as 20, 21, 29. Multiplying every number in a triple by the same whole number gives another triple.

How do I find a diagonal inside a box?

First find the diagonal of the base with the theorem, then use that diagonal and the height as the legs of a second right triangle. Equivalently, $d^2 = l^2 + w^2 + h^2$.

Why do I get a decimal for the answer?

Most sums of squares are not perfect squares, so the square root is irrational. Leave the answer as a root for an exact value, or round to the nearest tenth, as in $\sqrt{85} \approx 9.2$.