

Congruent and Similar Figures Through Dilations

Grade 8 math · Lesson · Standards: 8.G.A.2, 8.G.A.3, 8.G.A.4

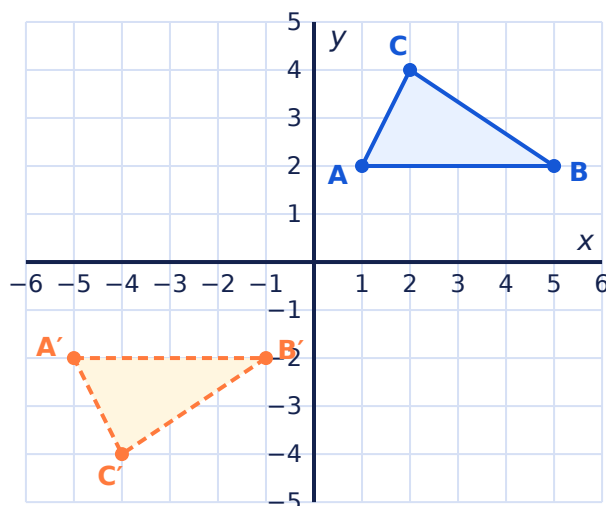
Quick answer: Congruent figures can be mapped onto each other by translations, reflections, and rotations. Similar figures need a dilation as well. A dilation about the origin multiplies every coordinate by the scale factor k , so lengths scale by k while angles stay the same.

Standards: 8.G.A.2, 8.G.A.3, 8.G.A.4 · **Level:** Grade 8

When are two figures congruent?

Congruent figures

Two figures are congruent if a sequence of translations, reflections, and rotations maps one exactly onto the other. Congruent figures have equal corresponding side lengths and equal corresponding angles.



Congruent triangles: a reflection followed by a translation maps $\triangle ABC$ onto $\triangle A'B'C'$.

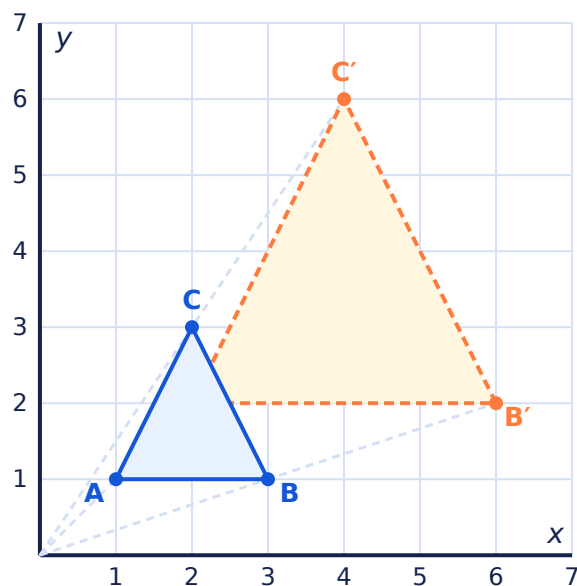
To prove congruence, name the moves in order. Here, the reflection flips the triangle across the x -axis and the translation then slides it 6 units left.

What does a dilation do?

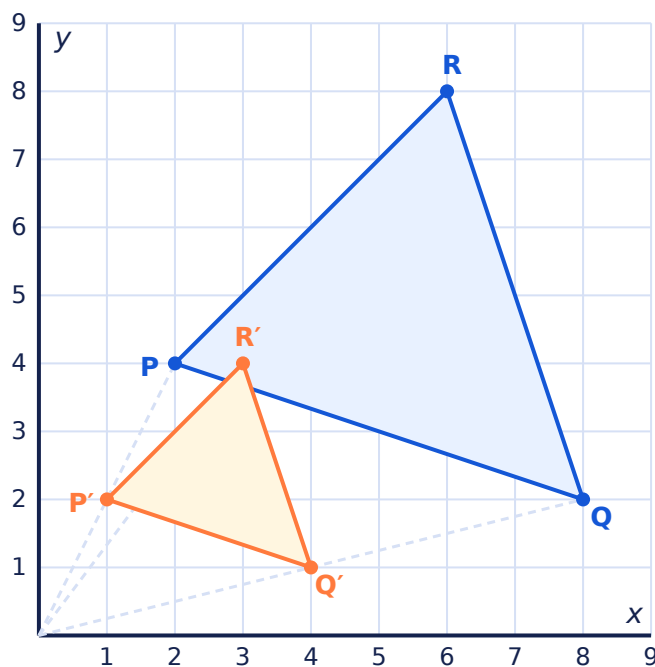
Dilation

A dilation about the origin with scale factor k multiplies both coordinates of every point by k .

The center stays fixed, each image point lies on the ray from the center through its pre-image, and every distance from the center is multiplied by k .



Dilation with center $(0, 0)$ and scale factor $k = 2$.



Dilation with scale factor $k = 1/2$ shrinks the figure toward the origin.

Dilation facts

- The rule is $(x, y) \rightarrow (kx, ky)$, with k greater than 0.
- Side lengths are multiplied by k ; angle measures do not change.
- A segment and its image are parallel.
- If k is greater than 1 the figure grows; if k is between 0 and 1 it shrinks.

How do you show that two figures are similar?

Similar figures

Two figures are similar if a sequence of dilations and rigid motions maps one onto the other. Corresponding angles are equal, and corresponding sides are proportional with ratio equal to the scale factor.

Method

1. Pick one pair of corresponding sides and divide the image length by the pre-image length to get k .
2. Check that every other pair gives the same ratio.
3. Describe the dilation first, then the translation, reflection, or rotation that lines up the figures.

Worked example

Dilate the segment from $M(4, -6)$ to $N(-2, 8)$ by $k = 1/2$. Then $M' = (2, -3)$ and $N' = (-1, 4)$. The length of MN is $\sqrt{232}$ and the length of $M'N'$ is $\sqrt{58}$. Since $\sqrt{232} = 2\sqrt{58}$, the image is half as long, as expected.

Common mistake

Adding the scale factor to the coordinates is wrong. Dilations multiply: with $k = 3$ the point $(2, 5)$ goes to $(6, 15)$, not $(5, 8)$.

Mathbot's tip

Congruent means $k = 1$ in disguise. If the ratio of corresponding sides is exactly 1, only rigid motions are needed.

Key points to remember

- Congruent figures are related by a sequence of rigid motions only.
- A dilation about the origin sends (x, y) to (kx, ky) .
- Similar figures have equal angles and proportional sides.
- The scale factor is image length divided by pre-image length.
- A scale factor of 1 changes nothing; below 1 shrinks, above 1 enlarges.

Congruent and Similar Figures Through Dilations: frequently asked questions

Can similar figures also be congruent?

Yes, when the scale factor is exactly 1. Every congruent pair is also similar with $k = 1$, but similar figures with a different scale factor are not congruent.

What is the center of a dilation?

The center is the one point that does not move. In this chapter it is always the origin, so each image point lies on the ray from $(0, 0)$ through its pre-image.

How do you find the scale factor from two figures?

Divide a side length of the image by the matching side length of the pre-image. For a 12-inch side that becomes 30 inches, $k = 30 \div 12 = 2.5$. Check a second pair of sides.

Do angles change under a dilation?

No. Angle measures are preserved, which is why dilated figures keep their shape. Only the lengths change, and they all change by the same factor.

What does a scale factor between 0 and 1 do?

It reduces the figure. Every point moves closer to the center, so a scale factor of $1/4$ makes each side one quarter as long and the image is a smaller copy of the same shape.